Probabilistic characterization of shear strain-dependency of shear modulus and damping ratio for Italian clays

Marco Uzielli,* Marco Zei,** Johann Facciorusso***

Summary

The quantitative modeling of the shear strain-dependency of shear modulus and damping ratio is a fundamental step in the characterization of the dynamic cyclic behavior of soils. Such dependency can be investigated directly through specialized cyclic dynamic laboratory testing on undisturbed soil samples and through the subsequent fitting of analytical models. This approach is difficult to implement in routine practice, and it is convenient to refer to empirical models relating dynamic soil properties to other geotechnical properties such as plasticity, which are known to influence dynamic behavior, and which are easily and routinely measured.

Like many technical and engineering disciplines, geotechnical engineering is experiencing a momentous shift in paradigm from the traditional deterministic approach to an uncertainty-based approach in which the inherent variability in datasets, uncertainties in measurements and engineering models used to transform in-situ and laboratory data to design parameters are modelled, processed, and reported explicitly, for instance through statistical and probabilistic techniques. An ever-increasing corpus of evidence from research and practice supports the argument that the uncertainty-based approach facilitates the improved understanding and engineering modelling of the unavoidable complexity in geotechnical systems, as well as cost-performance optimization in design and construction.

This paper presents a novel method for the probabilistic characterization of shear strain-dependency of shear modulus and damping ratio for clays of differing levels of plasticity. Simple analytical relationships, which only require the availability or estimate of the plasticity index and which are applicable for a wide range of probabilities of exceedance, are derived from a high-quality database of resonant column and cyclic torsional shear test results on 170 undisturbed clay specimens from Italian sites. The paper concludes by demonstrating how to apply the results practically in a simplified one-dimensional ground response analysis.

Keywords: cyclic dynamic behavior, shear modulus, damping ratio, statistics, probability, quantile regression

1. Introduction

Soil properties under cyclic and dynamic loading conditions are needed in the solution of most geotechnical earthquake engineering problems such as ground response analysis, dynamic soil-structure interaction, large-scale microzonation studies, and geotechnical design in the dynamic domain (*e.g.*, Abate and Massimino, 2016; Ansal *et al.*, 2010; Facciorusso *et al.*, 2016; Finn, 2000; Madiai *et al.*, 2017; Pagliaroli, 2018; Sanchez-Sesma *et al.*, 2002; Stokoe *et al.*, 1999). The most important properties influencing soil behavior in dynamic conditions are shear stiffness and damping, commonly expressed in terms of the shear modulus, G, and damping ratio, D, respectively. Both properties vary with the level of induced shear strain γ. Low-strain, intermediate-strain, and high-strain domains, involving increasingly inelastic, non-linear, dissipative, and cycle-dependent behavior, requiring distinct analysis approaches and modeling techniques, can be identified [Kramer, 1996], along with the linear and volumetric strain thresholds γ_l and γ_v which define the boundaries between the domains. For shear strain levels exceeding γ_l, the quantitative specification of the strain-dependent variation of both G and D with γ is required to account for nonlinear effects, for instance when applying an equivalent linear approach in seismic response analyses.

Earthquake geotechnical engineering analyses often require a site-scale or territorial-scale approach which entails the characterization of dynamic properties for many samples and locations. The direct modelling of shear-strain dependency, involving the fitting of analytical models to cyclic and dynamic testing measurements, would entail a considerable number of cyclic and dynamic laboratory tests, which may prove to be too expensive and time-consuming. Such drawback can be effectively circumvented through the availability of models allowing the parameterization of strain-dependency from outputs of routine geotechnical testing. The strain-dependency of G and D for fine-grained soils has been found to be mainly related to plasticity, with in-situ

* Dipartimento di Ingegneria Civile e Ambientale, Università degli Studi di Firenze, Georisk Engineering S.r.l., Italy
** Georisk Engineering S.r.l., Italy
*** Dipartimento di Ingegneria Civile e Ambientale, Università degli Studi di Firenze, Georisk Engineering S.r.l., Italy

 dx.doi.org/10.19199/2021.2.0557-1405.055

stress state, age and stress-strain history also playing an important role. Consequently, the plasticity index I_p (primarily), overconsolidation ratio *OCR*, void ratio *e*, confining stress σ'_0 and strain rate have been identified as efficient proxies [CAVALLARO *et al.*, 2003; DARENDELI, 2001; FACCIORUSSO and MADIAI, 2020; ISHIBASHI, 1992; SEED and IDRISS, 1970; VUCETIC and DOBRY, 1991; ZHANG *et al.*, 2005].

The economic and advantages resulting from the availability of empirical models are counterweighted by the fact that such models are developed from data from other sites, which almost invariably differ from each other and from the target site in terms of the properties which most influence the strain-dependency of shear modulus and damping ratio. The site-dependency of geotechnical properties and the merging of data from multiple sites introduces scatter in the databases used to derive such models. Such scatter increases the aleatory uncertainty stemming from the "real" variability of geotechnical properties within the database [UZIELLI *et al.*, 2007].

Geotechnical models have been traditionally developed using ordinary least-squares statistic regression. While this approach identifies a "central" tendency in the data and ensures the optimization of goodness-of-fit parameters such as root mean square error and coefficient of determination, it cannot duly describe the variability in the data set. Assessing such variability is of paramount importance in the forward utilization of geotechnical models stemming from a specific dataset. In the dynamic modeling of soil behavior for earthquake geotechnical engineering applications such as ground response analysis and seismic liquefaction triggering estimation, for instance, variability in values of G/G_0 (G_0 is the small-strain shear modulus) and *D* entail varying degrees of stiffness, dissipation of elastic energy and, ultimately, varying levels of amplification of seismic motion. While the complexity of ground response often prevents a reliable a-priori assessment of the effects of lower or higher shear modulus and damping ratio, it is important to assess the effects of the propagation of the uncertainty in the strain-dependency models.

This paper proposes simple relationships for the modeling of shear strain-dependency of shear modulus and damping ratio for fine-grained soils of varying degrees of plasticity and for user-defined levels of probabilities of exceedance of analytically derived values of shear modulus and damping ratio. The models only require the assignment of a plasticity category (based on available or hypothesized values of the plasticity index). The sequential process, involving the application of statistical quantile regression for and the subsequent analytical generalization of model coefficients, is described in detail. The output models are assessed and discussed critically in the light of available knowledge regarding strain-dependency of dynamic properties. Finally, the ease of applicability of the models is demonstrated in a simplified one-dimensional ground response analysis case-study application.

2. Source database and analyses dataset

The database used in the study includes results of resonant column (RC) and cyclic torsional shear (CTS) tests performed on 170 fine-grained specimens taken from 90 sites in Central and Northern Italy. CTS and RC tests were conducted on cylindrical specimens of 38.1 mm diameter and 76.2 mm length taken from undisturbed soil samples using a Stokoe-type, fixed-free equipment coupled with the cyclic torsional shear test apparatus. CTS and RC tests were performed on the same specimen, with CTS conducted first followed by RC for a given strain level (CTSRC test). Before applying the cyclic torsional loading, each of the tested specimens was saturated (*i.e.,* Skempton's coefficient *B* is checked to be greater than 0.95) and isotropically consolidated following one or two consolidation steps. Following CTS testing, the sample was generally reconsolidated at the same confining pressure of the first consolidation to dissipate residual excess pore water pressures.

It is worth noting that the main differences between RC and CTS tests lie in the range of frequency and the amplitude of excitations. CTS tests are static or quasi-static cyclic tests, while the frequencies of loading applied with RC tests lie between 10 and 100 Hz. In clayey specimens, measured values of *G* from RC tests are generally higher than those from CTS tests, with the difference increasing with the frequency employed in CTS tests [CAVALLARO *et al.*, 2003; D'ONOFRIO *et al.*, 1999; FEDRIZZI *et al.*, 2015]. Moreover, due to high values of resonance frequencies typically encountered (30-50 Hz), measurements of *D* from RC tests are usually higher than those obtained at the same strain levels from CTS tests where frequency loading is significantly lower [MONACO *et al.*, 2012; SANTUCCI DE MAGISTRIS *et al.*, 2013; SHIBUYA *et al.*, 1995], especially at high strain amplitudes. Also, when CTS and RC are performed sequentially on the same specimen, at shear strain levels exceeding the volumetric threshold (implying the build-up of high excess pore water pressure), specimens can exhibit higher values of G_0 following CTS testing.

The selected samples were taken mostly from Holocene and Pleistocene fluvio-lacustrine soil deposits at depths ranging from 1 m to 75 m below ground level. Samples mainly consist of normally consolidated and overconsolidated clayey silts or clays ($1 < OCR < 9.4$) of medium-to-high plasticity ($4 < I_p < 84$, with plasticity index I_p in %), with very low-to-high consistency index I_c ($-1 < I_c < 1.9$) and initial void ratio e_0 ranging between 0.175 and 2.456. Atterberg limits are plotted on the Casagrande plasticity chart in fi-

gure 1, while figure 2 plots initial void ratio against confining pressure. Table I summarizes the main physical, index properties and dynamic properties in terms of sample statistics, *i.e.*, minimum, maximum, mean, standard deviation, coefficient of variation, 10%, 50%, and 90% percentiles. *OCR* values were estimated from oedometric tests results when available. The database also includes five samples of organic clays of low consistency, very high water content, high void ratio, and low unit weight. Further details regarding the equipment used, the testing procedure adopted, and the database content and organization can be found in FACCIORUSSO [2020].

Some samples from the source database were not included in the dataset used for the analyses to increase the degree of homogeneity in dataset and, thus, the geotechnical significance and quality of the empirical models that can be obtained. As shown by many Authors [KALLIOGLOU, 2008; KISHIDA, 2009], organic soils exhibit a different behavior than inorganic soils (*e.g.*, in terms of dependency from the plasticity index), and were thus not included in the dataset used for the analyses. Moreover, an increasing level of homogeneity can be achieved through the subdivision of the source dataset into subsets according to an increasing number of parameters, it is advisable to preserve a sufficient level of generality to ensure the exportability of derived models beyond source sites. However, subsets should also be defined with the aim of ensuring the statistical significance of the analysis outputs by maintaining a sufficiently high sample numerosity. It is thus necessary to identify a tradeoff between dataset numerosity and geotechnical homogeneity. As mentioned previously, plasticity has been recognized as the primary governing factor in the shear-strain dependency of G and D [VUCETIC and DOBRY, 1991; DARENDELI, 2001]. The sub-categorization of the source database was thus performed according to intervals of I_p. More specifically, four plasticity categories were identified according to intervals of increasing values of I_p and superimposed on the Casagrande plasticity chart in figure 1 (dashed lines); namely: PC1 ($I_p \leq 15\%$), PC2 ($15\% < I_p \leq 25\%$), PC3 ($25\% < I_p \leq 35\%$), and PC4 ($35\% < I_p \leq 50\%$). Two samples with $I_p > 50\%$ and four samples where I_p was not available were not included in the dataset, which thus comprises 159 samples out of the total 170 in the source database.

Hereinafter, RC and CTS results are processed and analyzed together according to the aim of the paper, which is to provide a general model for the forward characterization of the shear strain-dependency of shear modulus and damping ratio for clays. The breadth and heterogeneity of the source dataset allows a more general application of the model on the part of the user, as the latter accommodates the aggregate variability of strain-dependency related to the inherent variability in soil behavior among different samples, measurement uncertainty, the differences between RC and CTS testing outputs due to distinct ranges of frequency of loading excitation, etc.

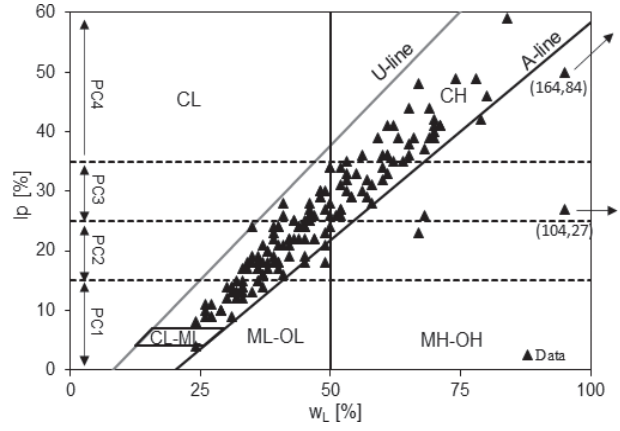


Fig. 1 – Classification of database samples and plotting of plasticity categories on the Casagrande chart.
Fig. 1 – Classificazione dei campioni del database e visualizzazione delle categorie di plasticità sulla carta di Casagrande.

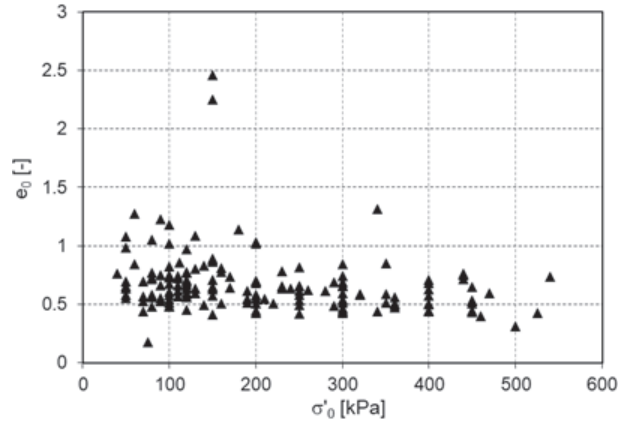


Fig. 2 – Initial void ratio versus effective confining pressures σ'_0 of database samples.
Fig. 2 – Indice dei vuoti iniziale e pressione efficace di confinamento σ'_0 dei campioni provenienti dal database.

Table II reports salient sample statistics for the four subsets corresponding to plasticity categories PC1-PC4 as defined above; namely: sample mean, minimum, and maximum of plasticity index; number of RC tests; number of CTS tests; sample numerosity (i.e., total number of measurements from RC and CTS tests pertaining to each category). The resulting sample numerosities for PC1-PC4 are deemed to be adequately balanced and ensure statistical significance for all categories.

3. Approach to probabilistic characterization

The probabilistic characterization of strain-dependency of shear modulus and damping ratio is con-

Tab. I – Sample statistics of complete source dataset (z = depth, G_s = specific gravity, γ = unit weight, w_L = liquid limit; I_P = plasticity index; w = water content, I_c = consistency index, e_0 = initial void ratio, OCR = overconsolidation ratio; G_0 = small-strain shear modulus; D_0 = small-strain damping ratio; γ_l = elastic shear strain threshold; γ_v = volumetric shear strain threshold).
Tab. I – Statistiche dei campioni del dataset di origine completo (z = profondità, G_s= peso specifico, γ = peso dell'unità di volume, w_L = limite liquido; I_P = indice di plasticità; w = contenuto in acqua, I_c = indice di consistenza, e_0= indice dei vuoti iniziale, OCR = rapporto di sovraconsolidazione; G_0= modulo di taglio a basse deformazioni; D_0 = rapporto di smorzamento a basse deformazioni; γ_l = soglia di deformazione lineare; γ_v = soglia di deformazione volumetrica).

Property	No. Obs	Min	Max	Mean	St.Dev.	CoV	10% perc.	50% perc.	90% perc.
z [m]	170	1.5	74.8	14.0	11.4	0.81	3.3	10.3	30.3
G_s [-]	170	2.32	2.87	2.70	0.06	0.02	2.65	2.71	2.75
γ [kN/m^3]	170	12.7	24.0	19.6	1.4	0.07	18.2	19.8	21.0
w_L [%]	165	24.0	164.0	47.6	16.8	0.35	31.0	45.0	68.0
I_P [%]	165	4.0	84.0	25.1	11.2	0.45	12.4	24.0	39.6
w [%]	170	8.5	129.8	27.6	12.5	0.45	18.9	25.4	37.8
I_c [-]	165	-1.0	1.9	0.7	0.4	0.51	0.3	0.8	1.1
σ'_0 [kPa]	170	40	600	214	127	0.59	80	190	400
e_0 [-]	164	0.175	2.456	0.670	0.262	0.39	0.448	0.631	0.874
OCR [-]	94	1.0	9.4	2.7	1.9	0.72	1.0	2.3	5.5
G_0 [MPa]	199	21	292	94	52	0.55	44	81	178
D_0 [%]	199	0.8	5.1	2.1	0.8	0.41	1.2	1.9	3.1
γ_l [%]	198	1.9E-04	2.3E-02	5.1E-03	3.5E-03	0.70	2.1E-03	4.3E-03	8.8E-03
γ_v [%]	164	7.7E-03	3.5E-01	5.2E-02	5.2E-02	1.02	1.8E-02	3.3E-02	1.1E-01

Tab. II – Sample statistics of subsets for plasticity categories PC1-PC4.
Tab. II – Statistiche dei campioni dei sottoinsiemi di dati relativi alle categorie di plasticità PC1-PC4.

Plasticity category	$I_{P,mean}$	$I_{P,min}$	$I_{P,max}$	$I_{P,std}$	$I_{P,cov}$	# RC tests	# CTS tests	Sample numerosity N
PC1	11.8	4	15	2.5	0.21	32	12	527
PC2	20.4	16	25	2.9	0.14	61	12	1122
PC3	29.4	26	35	2.9	0.10	43	6	780
PC4	42.5	36	84	9.2	0.22	29	9	493
all	25.0	4	84	10.8	0.43	165	39	2922

ducted here according to a three-step procedure involving: (1) selection of analytical models; (2) estimation of model coefficients for selected quantiles; and (3) analytical generalization of model coefficients. These steps are illustrated in detail in the following.

3.1. Model selection

Strain-dependency of dynamic soil properties can be pursued through a direct approach, involving the fitting of analytical models to data from cyclic testing. The first step of the probabilistic characterization process exploits the availability of the large database of RC and CTS testing outputs as detailed in Section 2. Among other models available to in the geotechnical literature (*e.g.*, RAMBERG and OSGOOD, 1943; KONDNER and ZELASCO, 1963; DARENDELI, 2001; DRNEVICH, 2017), the modified hyperbolic model by YOKOTA *et al.* [1981] is adopted here for the shear strain-dependency of shear modulus and damping ratio:

$$\frac{G}{G_0} = \frac{1}{1+\alpha_G \gamma^{\beta_G}} \tag{1}$$

$$\frac{D}{D_{max}} = \exp\left[-\lambda \cdot \left(\frac{G}{G_0}\right)\right] \tag{2}$$

in which γ is the shear strain, and α_G, β_G, D_{max} and λ are the model coefficients. Model coefficients D_{max} and λ are commonly determined through regression on experimental observations of D and G/G_0 from cyclic testing. However, this entails the implicit modeling of strain-dependency of damping ratio since

shear strain does not appear in equation (2). As RC and CTS tests allow the direct measurement of damping ratio for each level of shear strain, it is opportune to introduce shear strain into the exponential formulation of damping ratio given in equation (2) to obtain an explicit functional form. This is achieved by substituting equation (1) into equation (2) and rearranging terms:

$$D = D_{max} \cdot \exp\left[-\frac{\lambda}{1+\alpha_D \gamma^{\beta_D}}\right] \quad (3)$$

The set of model coefficients for the shear strain-dependency of damping ratio thus includes α_D, β_D, λ, and D_{max}. It is important to note that the approach adopted herein differs from the one originally proposed by YOKOTA *et al.*, in which the strain-dependency of damping ratio is given implicitly using equation (2) following the characterization of the strain-dependency of shear modulus. Moreover, the procedure adopted by YOKOTA *et al.* pertains in principle to a central tendency estimate for a single specimen, *i.e.*, in absence of inter-specimen variability. The model coefficients α_D and β_D in equation (3) should not be expected to coincide with the model coefficients α_G and β_G in equation (1) for several reasons. First, they derive from an aggregate dataset comprising multiple tests. Second, they are obtained directly through regression of damping ratio values from shear strain values. Third, they refer to multiple probabilities of exceedance, which are "inverse" for shear modulus and damping ratio: stiffer soils display lower values of damping ratio, and vice versa.

3.2. Empirical estimation of model coefficients

Once reference models for strain-dependency modeling of shear modulus and damping ratio have been established, it is possible to estimate model coefficients by performing statistical regression on data from each of the four plasticity categories defined from the source dataset.

Quantile regression is a type of regression analysiswhich is often used in statistics and econometrics, but which has received limited attention in geotechnical research so far. Applications are currently limited to UZIELLI and MAYNE [2019] and UZIELLI *et al.* [2019a; 2019b]. Whereas the method of least squares results in estimates that approximate the conditional mean of the response variable given certain values of the predictor variables, quantile regression aims at estimating either the conditional median or other quantiles of the response variable [YU *et al.*, 2003]. Quantile regression estimates are more robust against outliers in the response measurements relative to the ordinary least-squares regression. Consider the response model g *(i.e.*, the shear modulus reduction model in equation (1) or the damping ratio model in equation (2)):

$$\psi = g\left[\theta, \gamma\right] \quad (4)$$

in which ψ is an N-sized sample of the dependent variable (*i.e.*, G/G_0 or D) from the source dataset; γ is N-sized sample of independent variable (*i.e.*, shear strain); and θ is a vector of response model coefficients (*i.e.*, $\theta = [\alpha_G, \beta_G]$ for the $\gamma - G/G_0$ model; $\theta = [\alpha_D, \beta_D, D_{max}, \lambda]$ for the $\gamma - D$ - model). The regression quantile ω $(0<\omega<1)$ is defined as the solution to the quantile regression minimization problem

$$\sum_{i=1}^{N} f_{LP} \cdot \xi_i \quad (5)$$

where the loss function for the i-th value of ξ (i =1, …, N) is given by

$$f_{LP}(\xi_i) = \frac{|\xi_i| + (2\omega - 1)\xi_i}{2} \quad (6)$$

and

$$\xi_i = \psi_i - g(\theta_\omega, \gamma_i) \quad (7)$$

is the difference between the i-th database ψ_i value and model-predicted value for the i-th database value of shear strain γ_i. Quantile regression implements the minimization algorithm and yields estimates of model coefficients θ_ω pertaining to the regression quantile ω.

The empirical estimation of empirical coefficients of the shear-strain dependency models for shear modulus reduction (Eq. 1) and damping ratio (Eq. 2) was conducted for $\omega \in [0.05, 0.10, 0.25, 0.50, 0.75, 0.90, 0.95]$. The selected quantiles correspond, respectively, to probabilities of exceedance of 95%, 90%, 75%, 50%, 25%, 10%, and 5%.

3.3. Analytical generalization of model coefficients

In view of forward application, it is of interest to express model coefficients analytically for any quantile, beyond the set of quantiles used in the previous step of the modeling process. In the earthquake geotechnical engineering discipline, increasing use is made of probabilistic concepts. For instance, the analytical formulations of the deterministic seismic liquefaction resistance curve for use in the simplified methods based on cone penetration testing results proposed by BOULANGER and IDRISS [2014], MOSS *et al.* [2006] and JUANG *et al.* [2006] explicitly include the target probability of liquefaction. The generalization of model coefficients is pursued by fitting analytical functional forms to the YOKOTA *et al.* model coefficients obtained empirically for each dataset and for each quantile in the previous analysis. A trigonome-

Tab. III – Deterministic and probabilistic empirical model coefficients for shear-strain dependency of shear modulus reduction for selected quantiles by plasticity category.

Tab. III – Coefficienti empirici deterministici e probabilistici del modello di riduzione del modulo di taglio in funzione della deformazione di taglio, per quantili selezionati e per categoria di plasticità.

	PC1		PC2		PC3		PC4	
ω	α_G	β_G	α_G	β_G	α_G	β_G	α_G	β_G
det.	26.7	1.08	18.9	1.09	16.2	1.12	13.1	1.22
0.05	26.8	0.92	36.2	1.12	27.3	1.08	13.2	1.10
0.10	27.8	0.96	32.1	1.13	22.2	1.09	18.2	1.24
0.25	26.7	1.03	27.9	1.15	23.6	1.18	17.0	1.26
0.50	30.5	1.13	22.7	1.16	20.9	1.22	15.6	1.27
0.75	26.7	1.16	19.8	1.20	16.2	1.24	13.7	1.32
0.90	26.7	1.29	18.9	1.32	16.2	1.36	10.6	1.35
0.95	26.7	1.39	16.3	1.38	12.9	1.37	9.0	1.46

tric tangent model was adopted for all the model coefficients defining the strain-dependent behavior of shear modulus and damping ratio following a comparative assessment of the goodness of fit of several functional forms.

$$\theta_j = p_{1,\theta j} + p_{2,\theta j} \cdot \tan(p_{3,\theta j} \cdot \omega - p_{4,\theta j}) \qquad (8)$$

where θ_j denotes one of the model coefficients (α_G, β_G, α_D, β_D, D_{max}, or λ). The trigonometric model coefficients $p_{1,\theta j}$, $p_{2,\theta j}$, $p_{3,\theta j}$, and $p_{4,\theta j}$, are obtained through ordinary least squares regression on the respective sets of model coefficients obtained empirically by quantile regression as described above. The availability of analytical expressions allows generalization of the probabilistic definition of the shear strain-dependency of shear modulus reduction and damping ratio.

4. Modeling of strain-dependency of shear modulus

Regressions were conducted on each of the datasets pertaining to the 4 plasticity categories. Deterministic least-squares regression was also conducted for sake of comparison with quantile regression outputs. Table III summarizes the output empirical model coefficients α_G and β_G.

Figure 3 shows the shear modulus reduction models with coefficients tabulated in table III, resulting from the application of deterministic least-squares regression and quantile regression for the seven selected quantiles to data from plasticity categories PC1, PC2, PC3 and PC4.

Generalization of model coefficients was subsequently performed through regression of the quantile-related values given in table III (excluding the deterministic values) on the trigonometric model given in equation (8), yielding the coefficients in table IV. While inspection of table IV reveals that the coefficients p_{1,α_G}, p_{2,α_G}, p_{3,α_G}, and p_{4,β_G} take on virtually null values for plasticity category PC4, this does not occur for the other plasticity categories. Therefore, with the aim of providing a unique and consistent functional form for all categories, the general formulation of the model given in equation (8) is maintained for all plasticity categories. Figure 4 shows the fit of the trigonometric model to the two shear modulus model coefficients for PC2. The excellent fit of the trigonometric model to empirical model coefficients for α_G and β_G as given by the very high values of the adjusted coefficient of determination R^2_{adj} tabulated in table IV alongside the regressed coefficients attests to the quality of the outputs of the generalization process.

Figure 5 shows the modulus reduction curves obtained for the deterministic case (*i.e.*, using the coefficients given in table III) and example quantile values $\omega \in [0.15, 0.85]$ by applying equation (8) using the coefficients given in table IV for each of the plasticity categories PC1-PC4.

Figure 5 confirms how plasticity strongly influences modulus reduction curves and how more plastic soils exhibit a more rigid behavior with a broader elastic domain *(i.e.,* higher linear threshold strain values) and a more rapid reduction of G with increasing γ.

5. Modeling of strain-dependency of damping ratio

Applying deterministic regression and quantile regression to equation (9) yields a set of four coefficients (α_D, β_D, D_{max}, λ) for each reference quantile as given in table V.

Figure 6 shows the damping ratio models with coefficients tabulated in table V, resulting from the application of deterministic least-squares regres-

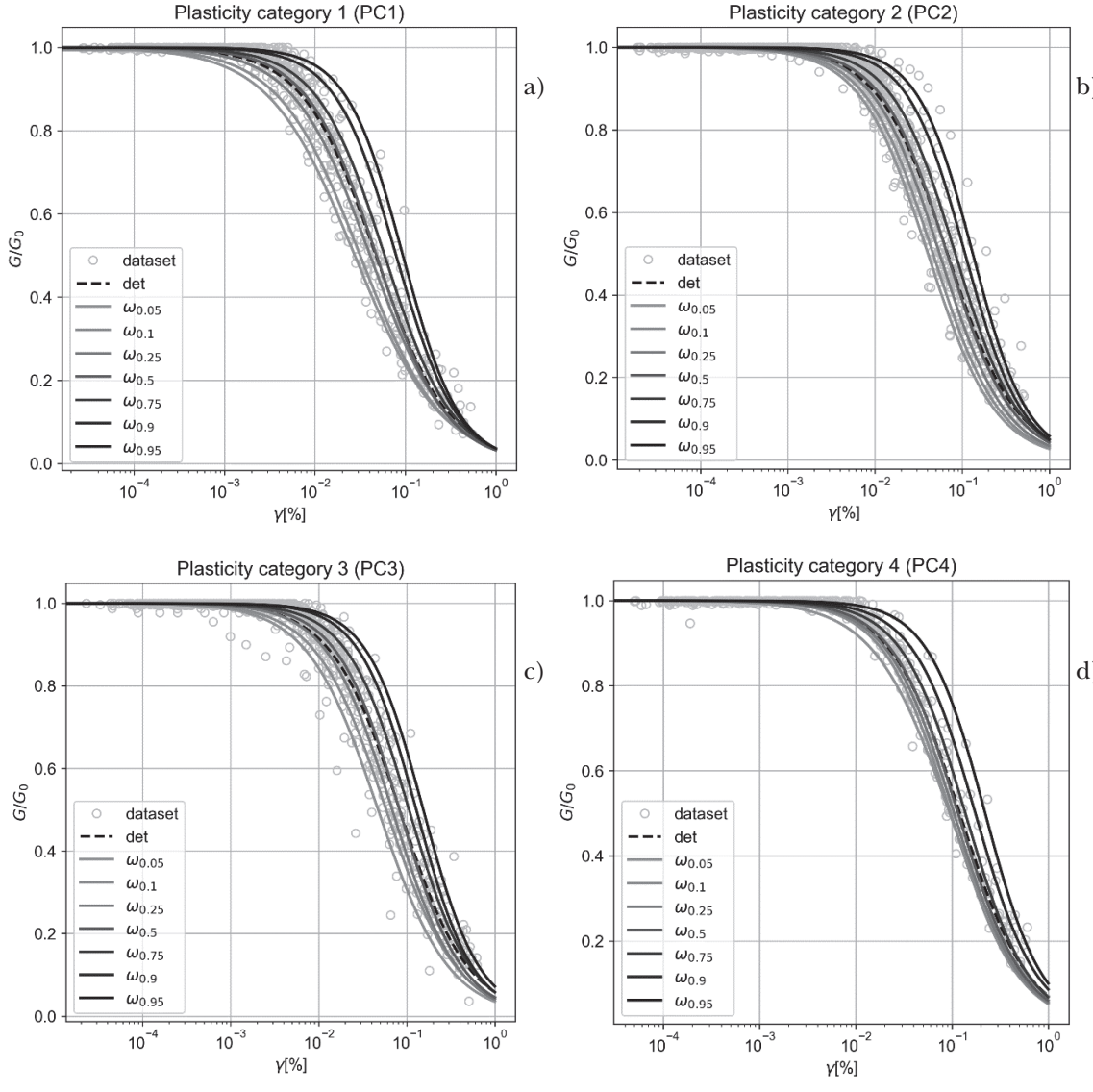


Fig. 3 – Deterministic and quantile regression-based models for shear modulus reduction for: a) PC1; b) PC2; c) PC3; d) PC4.
Fig. 3 – Modelli deterministici e derivanti dall'applicazione della regressione sui quantili per la stima della riduzione del modulo di taglio per: a) PC1; b) PC2; c) PC3; d) PC4.

sion and quantile regression for the seven selected quantiles to data from plasticity categories PC1, PC2, PC3 and PC4.

Generalization of model coefficients through regression of the trigonometric model in equation (8) on the quantile-related values given in table V yields the coefficients tabulated in table VI. Figure 7 shows the fit of the analytical trigonometric model to the four damping ratio model coefficients for PC2. As for shear modulus, the overall excellent fit of the trigonometric model to all sets of empirical model coefficients as parameterized by the generally very high values of the adjusted coefficients of determination R^2_{adj} tabulated in table VI attests to the reliability of the outputs of the generalization process.

Figure 8 shows the damping ratio curves obtained for the deterministic case (from Tab. V) and for $\omega \in [0.15, 0.50, 0.85]$ by applying equation (8) using the coefficients given in table VI for each of the plasticity categories PC1-PC4.

Figure 8 shows that damping ratio curves are significantly influenced by I_p. More specifically, at low strains and $\omega=[0.15,0.50]$, the damping ratio of lower-plasticity soils is smaller than that of higher-plasticity soils. As shear strain increases, less plastic soils exhibit a more dissipative behavior, and the damping ratio in these soils tends to increase more rapidly with increasing I_p than in more plastic soils. These observations were first presented in VUCETIC *et al.* [1998] and subsequently confirmed in other studies (*e.g.*, ZHANG *et al.*, 2005; FACCIORUSSO and MADIAI, 2020). For ω=0.85, a modest superposition between the curve for PC1 and the curves for PC2-PC3 is observed in the terminal upper ran-

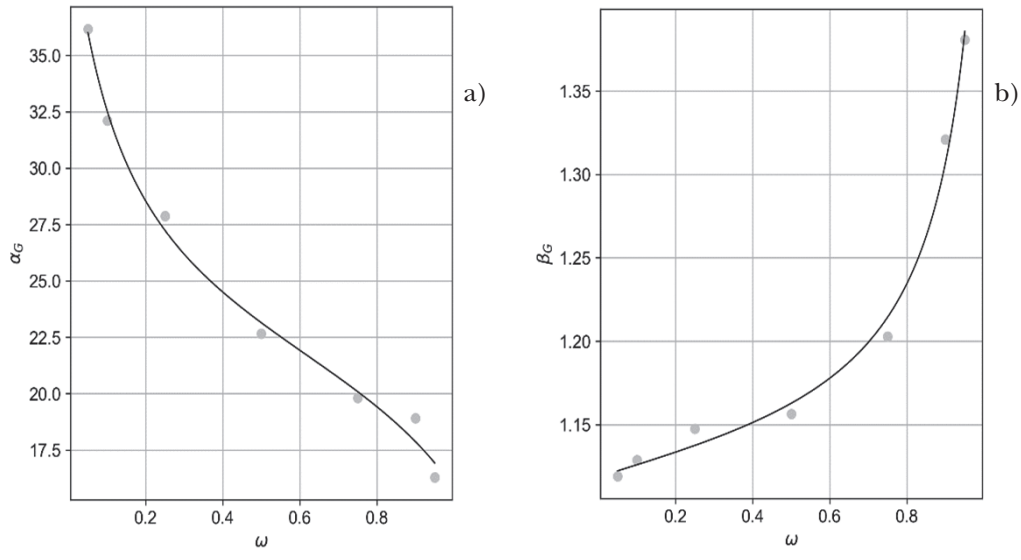


Fig. 4 – Analytical generalization of shear modulus model coefficients for plasticity category PC2: a) α_G, b) β_G.
Fig. 4 – Generalizzazione analitica dei coefficienti del modello per la riduzione del modulo di taglio per la categoria di plasticità PC2: a) α_G, b) β_G.

Tab. IV – Trigonometric model coefficients and adjusted coefficients of determination for the shear modulus reduction hyperbolic model coefficients α_G and β_G by plasticity category.
Tab. IV – Coefficienti del modello trigonometrico e coefficienti di determinazione aggiustati risultanti dalla stima dei coefficienti α_G and β_G del modello iperbolico di riduzione del modulo di taglio, per categoria di plasticità.

Coefficiente	PC1	PC2	PC3	PC4
$p_{1,\alpha G}$	27.77	21.73	20.12	16.42
$p_{2,\alpha G}$	-0.15	-5.66	-2.55	-1.34
$p_{3,\alpha G}$	1.53	2.11	2.67	1.47
$p_{4,\alpha G}$	0.00	1.30	1.31	0.00
R^2_{adj}	0.90	0.99	0.90	0.77
$p_{1,\beta G}$	1.10	1.12	1.21	1.29
$p_{2,\beta G}$	0.09	0.05	0.07	0.02
$p_{3,\beta G}$	2.68	1.46	2.49	3.30
$p_{4,\beta G}$	1.26	0.00	1.21	1.65
R^2_{adj}	0.99	0.99	0.97	0.99

ge of strain values, with the former intersecting the latter two. Such intersection can be attributed primarily to the arbitrary definition of plasticity categories and to the sensitivity of extreme quantiles to the scatter in the source dataset.

The probabilistic curves obtained herein are plotted in figure 9 against the deterministic curves obtained by DARENDELI [2001] for sake of comparison for PC2. It is remarked that the latter require the specification of plasticity index I_p, overconsolidation ratio *OCR*, and mean confining stress σ'_0. More specifically, shear modulus and damping ratio curves for ω =[0.10, 0.50, 0.90] are compared with the DARENDELI [2001] curves for I_p =20%, *OCR* =[1,8], and σ'_0 =[1,3] atm. The value I_p =20% corresponds to the central value of plasticity category PC2 as defined herein. Values of *OCR* and σ'_0 correspond approximately to the lower-bound and upper-bound values encountered in the source dataset for PC2. Source data points are also shown.

Inspection of figure 9 suggests that the DARENDELI [2001] models are not well-suited to represent the dataset used in this study. With respect to the median-quantile curve (ω=0.50) proposed herein, they generally underestimate shear modulus for $\gamma \leq 10^{-1.5}$ % for most combinations of *OCR* and σ'_0, while consistently overestimating shear modulus for greater values of shear strain. Conversely, they significantly underestimate damping ratio for $\gamma \leq 10^{-2}$ %. Similar observations can be made following comparative observations for PC1, PC3, and PC4 not shown here.

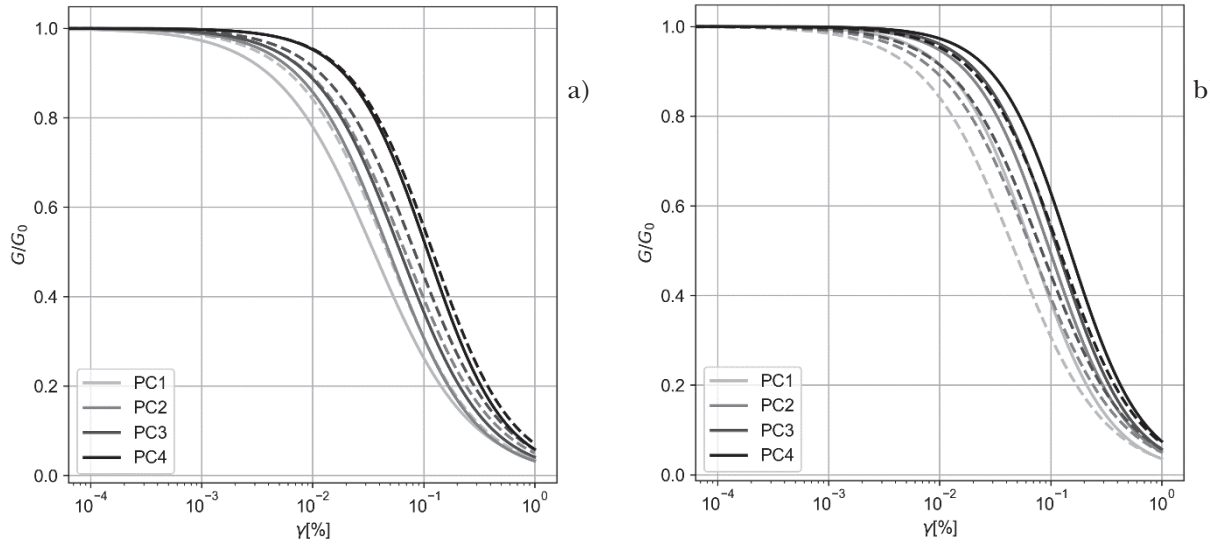


Fig. 5 – Comparison of deterministic (dashed lines) and probabilistic (solid lines) hyperbolic shear strain-dependency models for shear modulus reduction by plasticity category for: (a) ω =0.15; (b) ω=0.85.
Fig. 5 – Modelli iperbolici deterministici (linee tratteggiate) e probabilistici (linee continue) per la riduzione del modulo di taglio per categoria di plasticità per: (a) ω=0.15; (b) ω=0.85.

Tab. V – Deterministic and probabilistic empirical exponential model coefficients for shear-strain dependency of damping ratio for selected quantiles by plasticity category.
Tab. V – Coefficienti empirici deterministici e probabilistici del modello esponenziale di variazione del rapporto di smorzamento in funzione della deformazione di taglio, per quantili selezionati e per categoria di plasticità.

	PC1		PC2		PC3		PC4	
ω	α_D	β_D	α_D	β_D	α_D	β_D	α_D	β_D
det.	76.1	1.12	27.9	1.03	22.7	1.10	29.0	1.31
0.05	75.2	1.18	20.2	0.94	17.4	1.07	26.2	1.07
0.10	75.4	1.14	22.3	0.95	18.8	1.06	27.4	1.21
0.25	75.9	1.15	27.6	1.02	21.7	1.09	28.3	1.21
0.50	76.1	1.12	28.0	1.01	22.8	1.09	29.0	1.30
0.75	76.2	1.11	28.3	1.06	23.3	1.12	29.3	1.33
0.90	76.4	1.09	28.8	1.05	23.6	1.14	29.5	1.41
0.95	76.4	1.09	28.8	1.07	23.8	1.11	29.7	1.39
ω	D_{max}	λ	D_{max}	λ	D_{max}	λ	D_{max}	λ
det.	18.1	2.43	19.3	2.17	18.8	1.83	16.2	1.78
0.05	14.6	2.69	12.7	2.38	12.3	1.94	8.6	1.66
0.10	15.0	2.65	13.9	2.36	13.4	1.94	11.5	1.80
0.25	16.6	2.57	18.8	2.46	16.5	2.00	12.9	1.78
0.50	18.3	2.50	19.7	2.29	19.6	1.96	16.2	1.83
0.75	19.7	2.38	21.2	2.05	21.5	1.83	18.6	1.80
0.90	21.6	2.29	23.2	2.01	23.4	1.61	21.1	1.71
0.95	21.6	2.18	23.6	1.81	24.1	1.48	22.0	1.64

6. Example application to 1D ground response analysis

The proposed method is applied to a simplified one-dimensional ground response analysis to provide a quantitative example of the sensitivity of predicted seismic response effects to varying probabilities of exceedance in the modeling of shear strain-dependent shear modulus and damping ratio. A horizontally layered soil column of 30 m height overlying a half-space bedrock was considered. The soil column was discretized into seven sub-layers, with shear wave velocity increasing with depth, with a 30-meter average value V_s=230

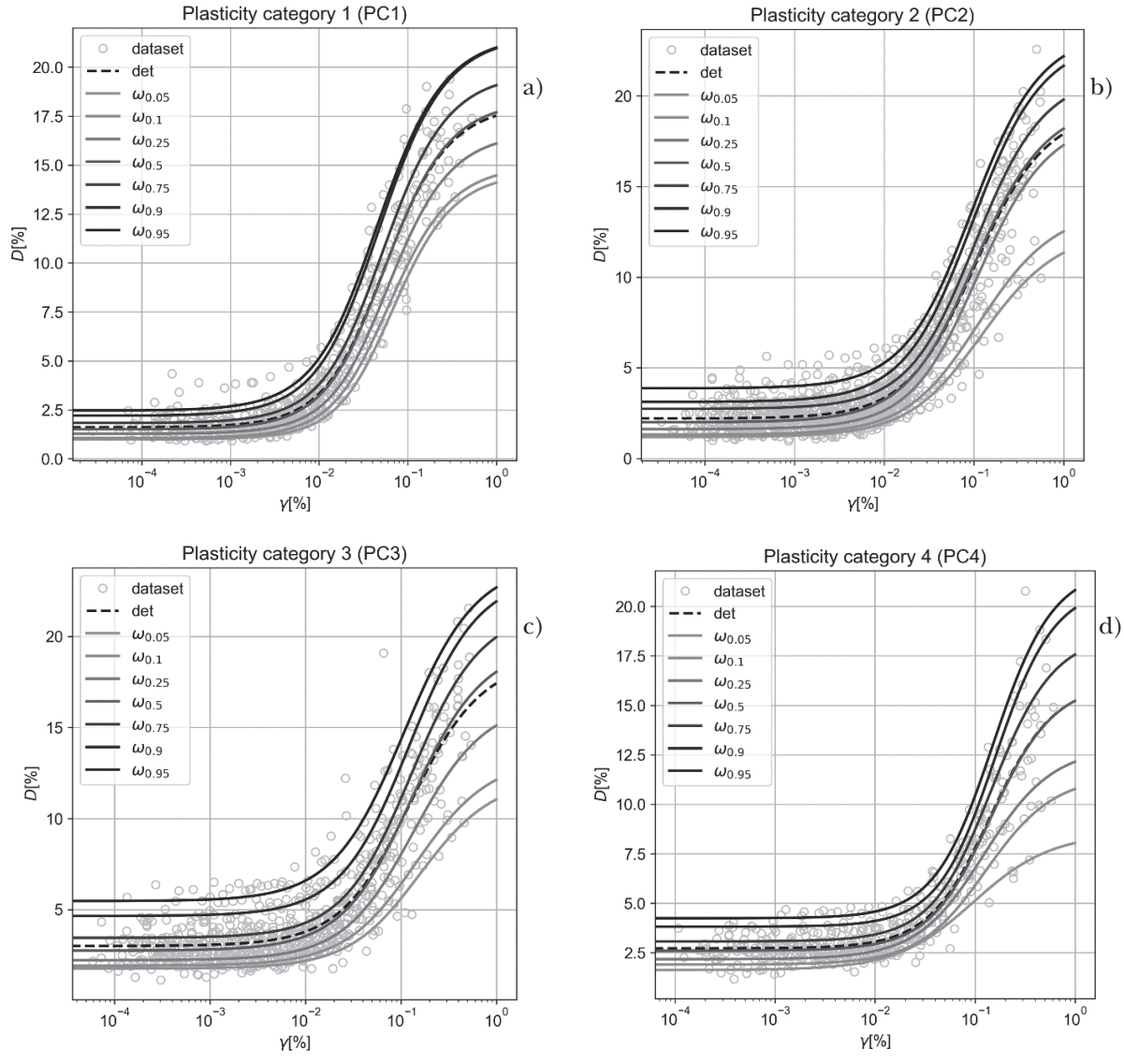


Fig. 6 – Deterministic and quantile regression-based exponential models for damping ratio for: a) PC1; b) PC2; c) PC3; d) PC4.

Fig. 6 – Modelli deterministici e probabilistici esponenziali di variazione del rapporto di smorzamento derivanti dall'applicazione della regressione sui quantili per: a) PC1; b) PC2; c) PC3; d) PC4.

m/s. The stratigraphic model was adopted alternately for each of the four plasticity categories PC1-PC4. Thus, for each modeling instance, the seven soil sub-layers were modelled using the same shear strain-dependency curves for both the modulus reduction and the damping ratio. The bedrock was assumed to be elastic with shear wave velocity of 800 m/s. In selecting reference quantiles for geotechnical applications, reference quantiles were defined for shear modulus as ω_G and for damping ratio as $\omega_D = 1 - \omega_G$. In the example presented herein, three values of ω_G [0.10, 0.50, 0.90] were considered, corresponding to ω_D =[0.90, 0.50, 0.10], respectively. The adopted relationship between ω_G and ω_D is not the only possible one, and other relationships can be defined, provided that they are consistent with fundamental soil dynamics knowledge and experience. Bedrock and soil layer properties are summarized in table VII.

Linear equivalent analyses were performed in the frequency domain and in terms of total stresses using the free software STRATA [KOTTKE and RATHJE, 2009]. Single Ricker waves (Ricker Wavelets Pulse, RWP) were applied as input motions at the bedrock. RWPs were modeled using two distinct frequency values $f_R \in [1.2,10$ Hz] and amplitude 0.79g. Figure 10 shows the stratigraphic model along with the acceleration time-histories of the two loading patterns.

Outputs of 1D numerical analyses were expressed in terms of: (a) acceleration transfer function *ATF (i.e.*, the ratio of the Fourier Amplitude Spectrum of the acceleration between the soil deposit at

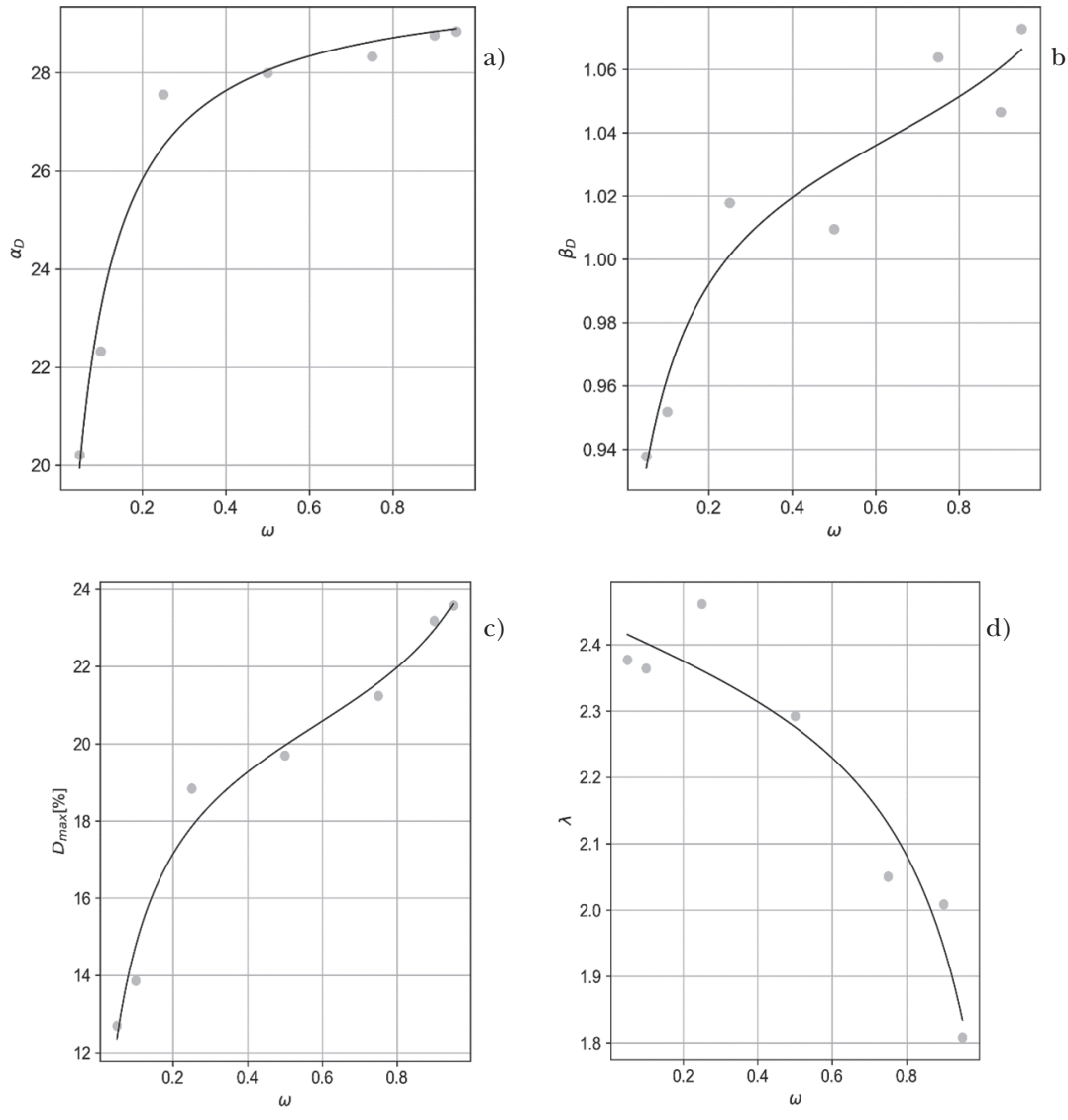


Fig. 7 – Analytical generalization of damping ratio exponential model coefficients for plasticity category PC2: a) α_D; b) β_D; c) D_{max}; d) λ.
Fig. 7 – Generalizzazione analitica dei coefficienti del modello esponenziale di variazione del rapporto di smorzamento per la categoria di plasticità PC2: a) α_D; b) β_D; c) D_{max}; d) λ.

ground surface and outcropping bedrock); (b) peak ground acceleration *PGA*; and (c) the maximum value of shear strain γ_{max} attained during ground shaking. Examples of these outputs are plotted in figures 11, 12 and 13, respectively. In terms of notation, outputs are presented by referring to the value of ω_G.

Figure 11, which plots *ATF* from deterministic and probabilistic analyses by plasticity category for f_R =10.0 Hz, shows that peak values of *ATF* decrease while fundamental periods *(i.e.,* corresponding to peak *ATF* values) increase with increasing plasticity. The progression of peak *ATF* with ω_G is not monotonic, with maximum peak values occurring for ω_G =0.50 for all plasticity categories. Deterministic transfer functions generally plot close to the median (ω_G =0.50) functions for all plasticity categories. Outputs such as the non-monotonic relationship between ω_G and peak values of *ATF* cannot in principle be predicted a-priori, as they depend on the relative influence of concurring effects of stiffness and hysteresis.

Figure 12 plots depth-wise profiles of *PGA* by plasticity category obtained from deterministic and probabilistic strain-dependency models for Ricker wavelet input frequency f_R =10.0 Hz. Peak acceleration at ground surface and shallow depths increases with increasing plasticity, with the effect of plasticity progressively decreasing with depth. The scatter between profiles pertaining to different values of ω_G decreases with increasing plasticity. Similarly to *ATF*, deterministic profiles generally plot close to the median probability profile, which also provides the hi-

Tab. VI – Trigonometric model coefficients and adjusted coefficients of determination for the exponential damping ratio model coefficients α_D; β_D; D_{max}; and λ. by plasticity category.

Tab. VI – Coefficienti del modello trigonometrico coefficienti di determinazione aggiustati risultanti dalla stima dei coefficienti α_D; β_D; D_{max}; and λ. del modello esponenziale di variazione del rapporto di smorzamento, per categoria di plasticità.

Coefficient	PC1	PC2	PC3	PC4
p_{1,α_D}	76.50	29.94	24.89	29.86
p_{2,α_D}	0.32	0.01	0.01	0.52
p_{3,α_D}	1.00	0.0	0.0	1.0
p_{4,α_D}	1.39	1.57	1.57	1.48
R^2_{adj}	0.99	0.97	1.00	0.98
p_{1,β_D}	1.12	1.04	1.33	1.30
p_{2,β_D}	-0.01	0.03	0.00	0.05
p_{3,β_D}	2.93	2.1	0.0	2.8
p_{4,β_D}	4.65	1.36	1.56	1.50
R^2_{adj}	0.95	0.92	0.82	0.94
$p_{1,D_{max}}$	18.32	20.46	20.47	16.36
$p_{2,D_{max}}$	3.54	2.63	5.21	3.97
$p_{3,D_{max}}$	1.78	2.4	1.8	2.3
$p_{4,D_{max}}$	0.91	1.38	1.10	1.19
R^2_{adj}	0.99	0.98	1.00	0.99
$p_{1,\lambda}$	2.50	2.43	1.99	1.77
$p_{2,\lambda}$	-0.12	-0.20	-0.08	0.00
$p_{3,\lambda}$	2.46	1.3	1.5	1.6
$p_{4,\lambda}$	1.12	0.00	0.00	0.00
R^2_{adj}	1.00	0.93	0.96	0.42

Tab. VII – Bedrock and soil layer properties employed in the example 1D response analysis.

Tab. VII – Proprietà del substrato e del deposito di terreno utilizzate nell'esempio di risposta sismica monodimensionale.

Depth [m]	**Thickness [m]**	**[kN/m^3]**	**[m/s]**
0.0	2.5	19.0	100
2.5	2.5	19.0	150
5.0	5.0	19.0	200
10.0	5.0	19.0	250
15.0	5.0	19.0	300
20.0	5.0	19.0	350
25.0	5.0	19.0	400
30	half-space	19	800

ghest values of *PGA* among the three quantiles considered in the example.

Figure 13 plots the depth-wise profile of maximum shear strain γ_{max} by plasticity category obtained from deterministic and probabilistic strain-dependency models for Ricker wavelet input frequency f_R =10 Hz. Values of γ_{max} decrease with increasing plasticity, as well as the scatter between outputs from probabilistic models at different quantiles. Deterministic outputs plot very close to median probability outputs in all cases. As in the previous figures, the non-monotonic relationship between γ_{max} and ω_G is visible, with the median profiles displaying the highest values and values from ω_G =0.10 exceeding those from ω_G =0.90 at most depths for all plasticity categories.

7. Discussion of results and concluding remarks

The strain-dependent curves for shear modulus reduction and damping ratio proposed in the study effectively capture the aggregate geotechnical uncertainty in the source database stemming mainly from inter-site and inter-sample variability. The implementation of statistical regression in

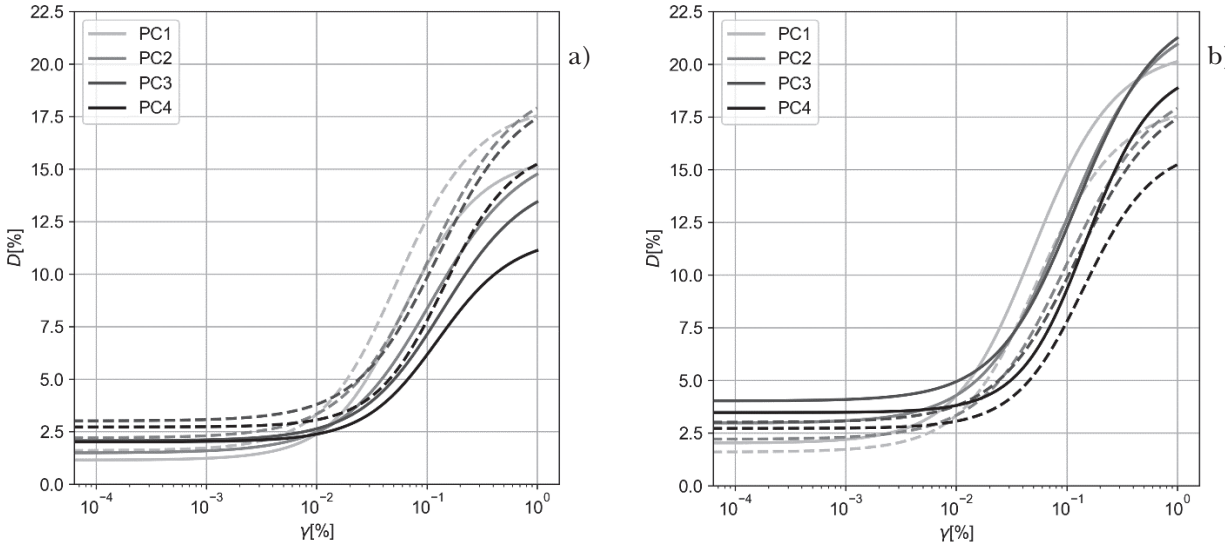


Fig. 8 – Comparison of deterministic (dashed lines) and probabilistic (solid lines) exponential shear strain-dependency models for damping ratio by plasticity category for: a) =0.15; b) =0.85.

Fig. 8 – Modelli deterministici (linee tratteggiate) e probabilistici (linee continue) esponenziali di variazione del rapporto di smorzamento per categoria di plasticità per: a) =0.15; b) =0.85.

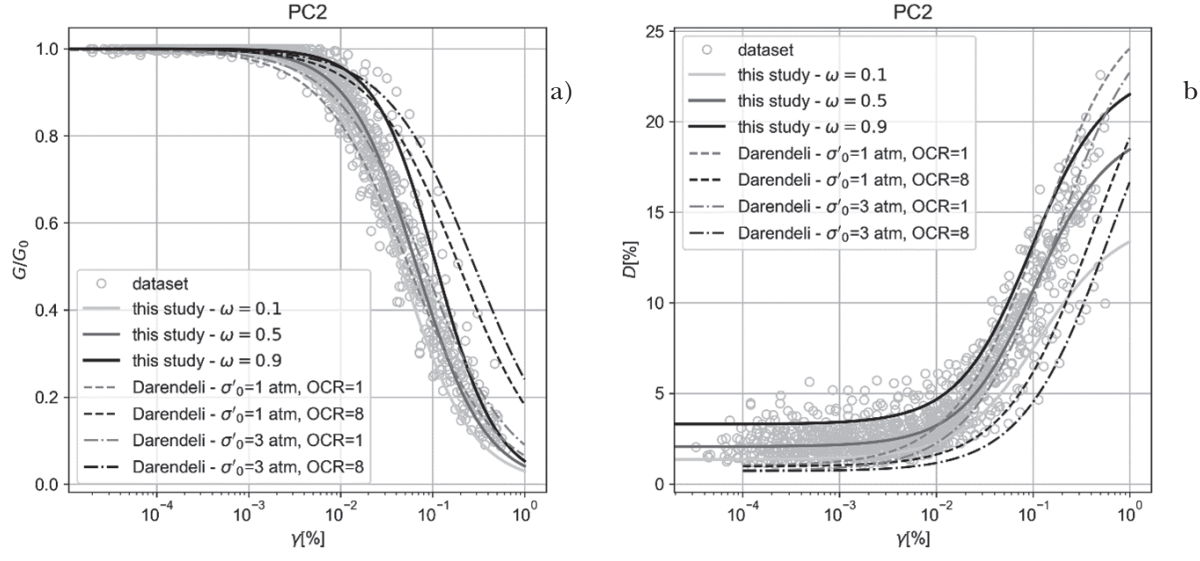


Fig. 9 – Comparison of the outputs of probabilistic modeling for =[0.10, 0.50, 0.90] and deterministic curves from DARENDELI [2001] for plasticity category PC2: a) shear modulus; b) damping ratio.

Fig. 9 – Confronto tra i modelli probabilistici ottenuti nel presente studio per =[0.10, 0.50, 0.90] e le curve deterministiche ottenute da DARENDELI [2001] per la categoria di plasticità PC2: a) modulo di taglio; b) rapporto di smorzamento.

the derivation of empirical model coefficient entails that all sources of aleatory and epistemic uncertainties are implicitly accounted for. The progression of probabilistic curves with increasing quantile is overall very consistent with well-established knowledge and literature findings, as well as important phenomena such as the rank inversion of damping ratio curves pertaining to different plasticity levels at intermediate-to-high-strain levels.

The simple analytical models presented herein can be updated upon availability of new data. The effects of increased sample numerosity in terms of aggregate uncertainty cannot be assessed with certainty a priori: new data may increase or reduce the "real" intra- and inter-site scatter, *i.e.*, aleatory uncertainty. Statistical estimation uncertainty reduces with increasing data numerosity, while the magnitude of measurement uncertainty depends on the quality of testing instrumentation and procedures. Plasticity ca-

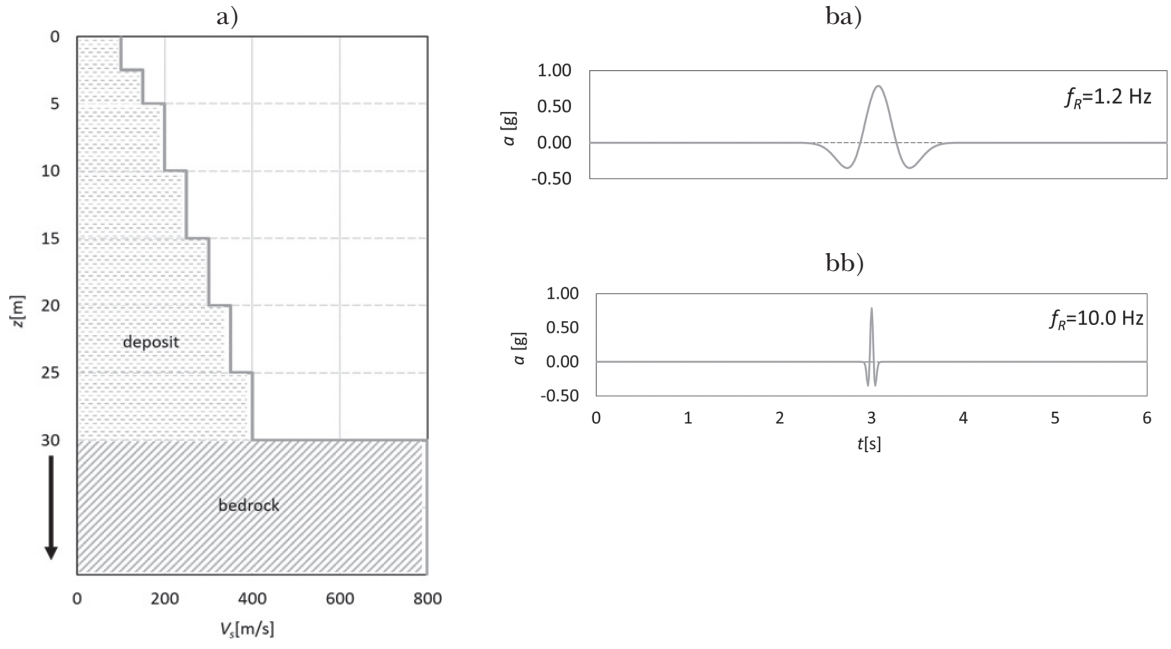


Fig. 10 – Example 1D ground response application: a) stratigraphic and shear wave velocity profiles; b) Ricker wavelet inputs.
Fig. 10 – Esempio di risposta monodimensionale: a) profilo stratigrafico e profilo della velocità delle onde di taglio; b) ondine di Ricker.

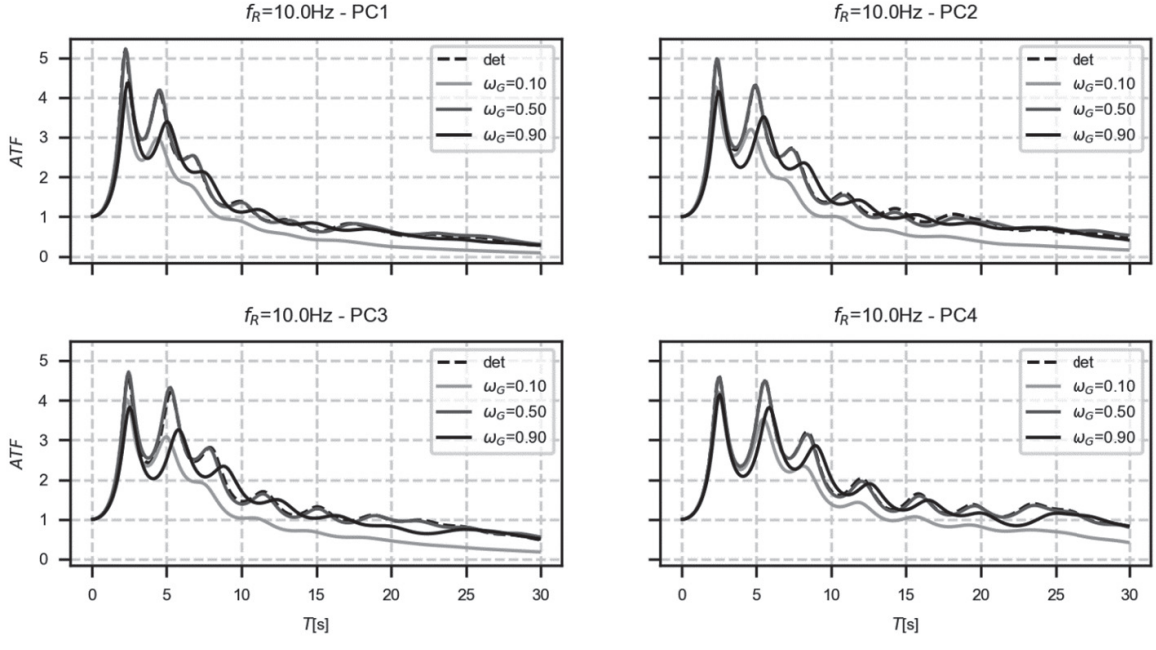


Fig. 11 – Transfer functions by plasticity category obtained from deterministic and probabilistic strain-dependency models (Ricker wavelet input frequency =10.0 Hz).
Fig. 11 – Funzioni di trasferimento ottenute da modelli deterministici e probabilistici di (frequenza delle ondine di Ricker =10.0 Hz).

tegories may also be redefined if necessary and opportune, depending on the future numerosity of the respective subsets.

While the simple one-dimensional ground response example presented herein is not aimed at the exhaustive and systematic parametric analysis of the effects of seismic loading frequency, soil plasticity and quantile, the pronounced variability in outputs attest to the inherent complexity of seismic response and to its sensitivity to user inputs in the modeling of shear strain-dependency. In this regard, the availability of probabilistic models allows an enhanced comprehension of the sensitivity of model outputs and a rational assessment of the effects of the propaga-

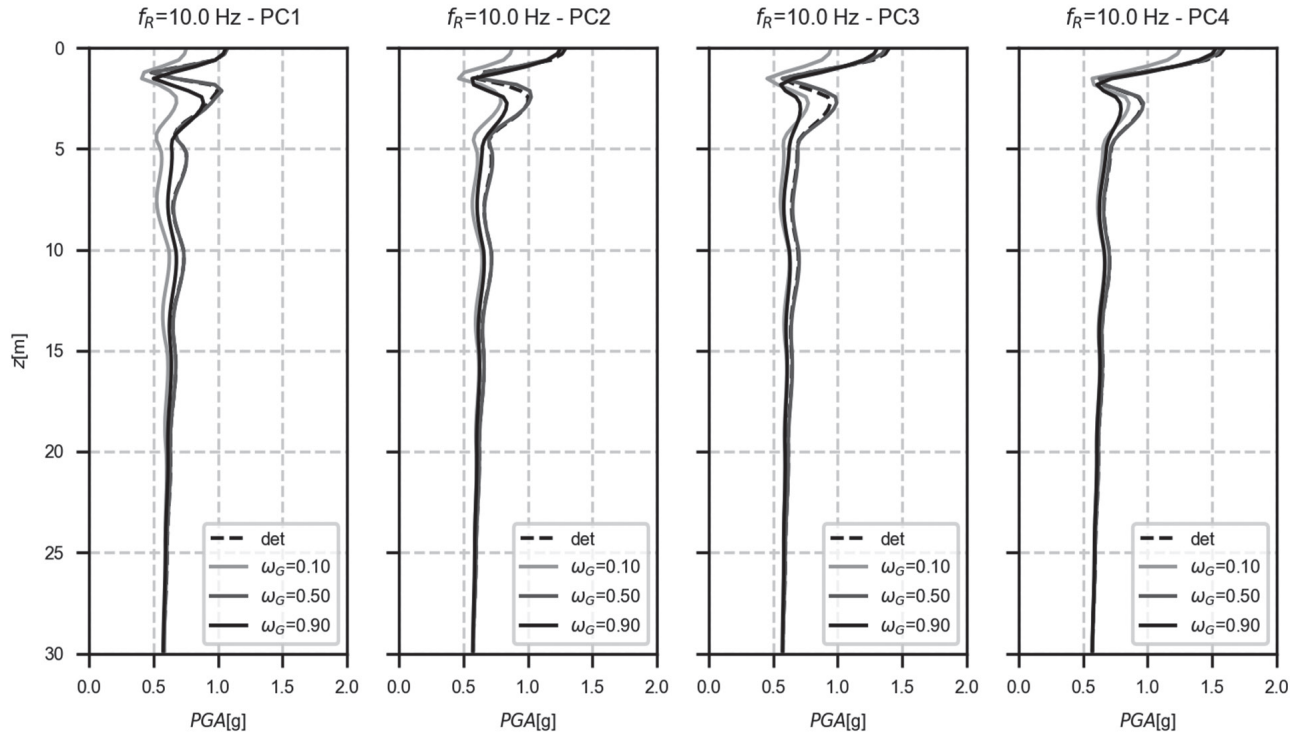


Fig. 12 – Depth-wise profile of peak ground acceleration by plasticity category obtained from deterministic and probabilistic strain-dependency models (Ricker wavelet input frequency =10.0 Hz).
Fig. 12 – Profili di variazione dell'accelerazione di picco con la profondità ottenuti da modelli deterministici e probabilistici (frequenza delle ondine di Ricker =10.0 Hz).

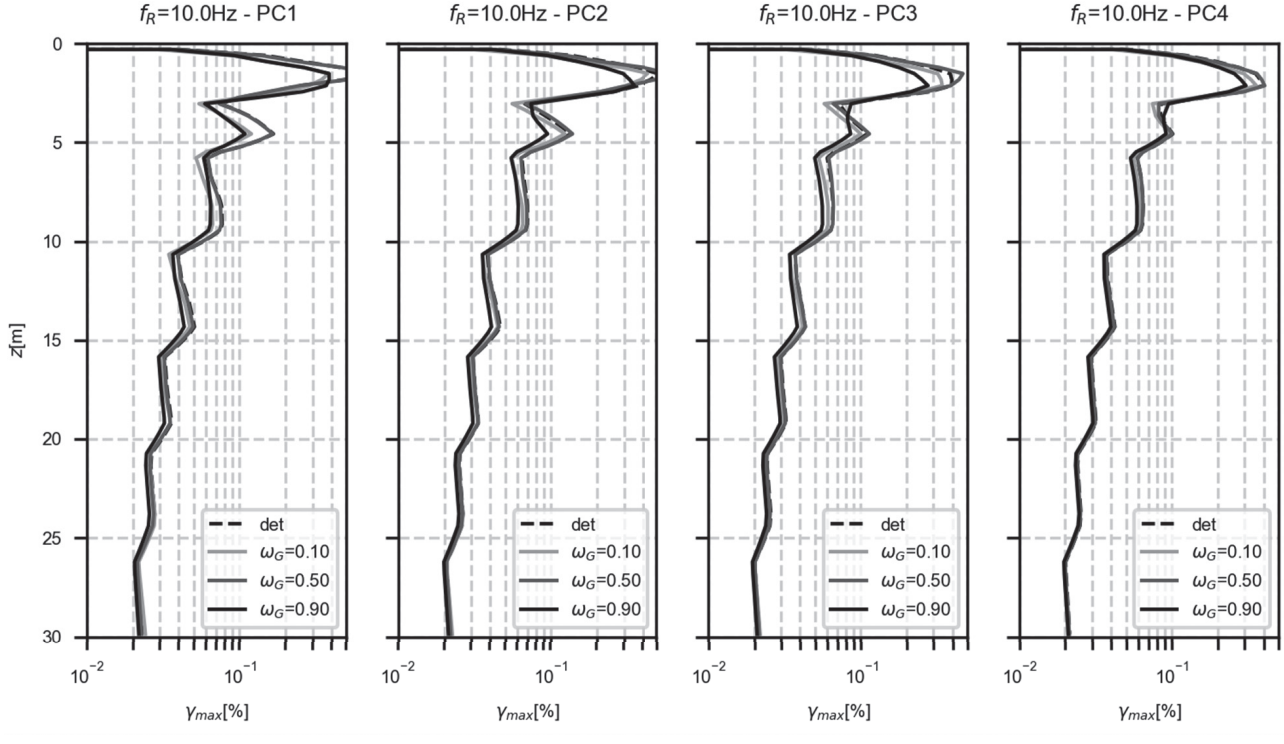


Fig. 13 – Depth-wise profile of maximum shear strain by plasticity category obtained from deterministic and probabilistic strain-dependency models (Ricker wavelet input frequency =10 Hz).
Fig. 13 – Profili di variazione della massima deformazione di taglio con la profondità ottenuti da modelli deterministici e probabilistici (frequenza delle ondine di Ricker =10 Hz).

tion of the inherent variability in data-driven strain-dependency models from the attributes of a specific source dataset.

Acknowledgements

The Authors would like to acknowledge the members of the Geotechnical Group of the Department of Civil and Environmental Engineering, and the Geotechnical Laboratory technicians at the University of Florence for their contributions to the formation and validation of the cyclic testing database used in the study.

References

ABATE G., MASSIMINO M.R. (2016) – *Dynamic soil-structure interaction analysis by experimental and numerical modelling*. Rivista Italiana di Geotecnica, n. 2, pp. 44-70.

ANSAL A., KURTULUŞ A., TÖNÜK G. (2010) – *Seismic microzonation and earthquake damage scenarios for urban areas*. Soil Dynamics and Earthquake Engineering, n. 30, pp. 1319-1328.

BOULANGER R.W., IDRISS I.M. (2014) – *CPT and SPT Based Liquefaction Triggering Procedures*. 1023 UCD/CGM-14/01, April 2014.

CAVALLARO A., LANZO G., PAGLIAROLI A., MAUGERI M., LO PRESTI D. WG. (2003) – *A comparative study on shear modulus and damping ratio of cohesive soil from laboratory tests*. Proceedings of the 3rd International Symposium on Deformation Characterisitics of Geomaterials, pp. 257-265.

DARENDELI M.B. (2001) – *Development of a new family of normalized modulus reduction and material damping curves*. Ph.D. Thesis, University of Texas, Austin, U.S.

DRNEVICH V.P. (2017) – *A Relationship between Modulus and Damping Provides for Simple, Unified Modeling of Both*. Geotechnical Frontiers, American Society of Civil Engineers, pp. 48-60.

D'ONOFRIO A., SILVESTRI F., VINALE F. (1999) – *Strain-rate dependent behaviour of a natural stiff clay*. Soils and Foundations, n. 39, pp. 69-82.

FACCIORUSSO J. (2020) – *An archive of data from Resonant Column and Cyclic Torsional Shear Tests performed on Italian Clays*. Earthquake Spectra, 37, n. 1, pp. 545-562.

FACCIORUSSO J., MADIAI C. (2020) – *On Cohesive Soil Damping Estimation by Free Vibration Method in Resonant Column Test*. Geotechnical Testing Journal, 43, n. 6, pp.1499-1518.

FACCIORUSSO J., MADIAI C., VANNUCCHI G. (2016) – *The 2012 Emilia earthquake (Italy): Geotechnical characterization and ground response analyses of the paleo-Reno river levees*. Soil Dynamics and Earthquake Eng., 86, pp. 71-88.

FEDRIZZI F., RAVIOLO P.L., VIGANÒ A. (2015) – *Resonant column and cyclic torsional shear experiments on soils of the Trentino valleys (NE Italy)*. Geotechnical Engineering for Infrastructure and Development, pp. 3437-3442.

FINN W.D.L. (2000) – *State-of-the-art of geotechnical earthquake engineering practice*. Soil Dynamics and Earthquake Eng., 86, pp. 71-88.

ISHIBASHI I. (1992) – *Discussion to "Effect of soil plasticity on cyclic response" by M. Vucetic and R. Dobry*. Journal of Geotechnical Engineering ASCE, n. 118, pp. 830-832.

JUANG C.H., FANG S.Y., KHOR E.H. (2006) – *First-order reliability method for probabilistic liquefaction triggering analysis using CPT*. Journal of Geotechnical and Geoenvironmental Engineering, 132, n. 73, pp. 337-350.

KALLIOGLOU P., TIKA T., KONINIS G., PAPADOPOULOS S., PITILAKIS K. (2009) – *Shear Modulus and Damping Ratio of Organic Soils*. Geotech. Geol. Eng., 27, pp. 217-235.

KISHIDA T., BOULANGER R.W., ABRAHAMSON N.A., WEHLING T.M., DRILLER M.W. (2009) – *Regression models for dynamic properties of highly organic soils*. Journal of Geotechnical and Geoenvironmental Engineering, 135, n. 4, pp. 533-543.

KONDNER R. L., ZELASKO J.S. (1963) – *A hyperbolic stress-strain formulation of sands*. Proc. 2nd Pan Am. Conf. on Soil Mech. And Found. Engrg., Brazilian Association of Soil Mechanics, pp. 289-324.

KOTTKE A.R., RATHJE E.M. (2009) – *Technical manual for Strata*. PEER Report, 2008/10.

KRAMER S.L. (1996) – *Geotechnical Earthquake Engineering*, 1st edition, Prentice Hall, Upper Saddle River, New Jersey, 673 p.

MADIAI C., FACCIORUSSO J., GARGINI E. (2017) – *Numerical modeling of seismic site effects in a shallow alluvial basin of the Northern Apennines (Italy)*. Bulletin of the Seismological Society of America, n. 107, pp. 2094-2105.

TOTANI G., BARLA G., CAVALLARO A., COSTANZO A., D'ONOFRIO A., EVANGELISTA A., FOTI S., GRASSO S., LANZO G., MADIAI C., MARASCHINI M., MARCHETTI S., MAUGERI M., PAGLIAROLI A., PALLARA O., PENNA A., SACCENTI A., SANTUCCI DE MAGISTRIS F., SCASSERRA G., SILVESTRI F., SIMONELLI A.L., SIMONI G., TOMMASI P., VANNUCCHI G., VERRUCCI L. (2012) – *Geotechnical Aspects of the L'Aquila Earthquake*. Special Topics in Earthquake Geotechnical Engineering, Springer, Dordrecht, pp. 1-66.

MOSS R.E.S., SEED R.B., KAYEN R.E., STEWART J.P., DER KIUREGHIAN A., CETIN K.O. (2006) – *CPT-based probabilistic and deterministic Assessment of in situ sesimic soil liquefaction potential*. Journal of Geotechnical and Geoenvironmental Engineering, 132, n. 8, pp. 1032-1051.

PAGLIAROLI A. (2018) – *Key issues in Seismic Microzonation studies: Lessons from recent experiences in Italy*. Rivista Italiana di Geotecnica, n. 1, pp. 5-48.

RAMBERG W., OSGOOD W.R. (1943) – *Description of*

stress-strain curves by three parameters. Technical Note No. 902, National Advisory Committee for Aeronautics, Washington, D.C., USA.

SÁNCHEZ-SESMA F.J., PALENCIA V.J., LUZÓN F. (2002) – *Estimation of local site effects during earthquakes: an overview*. Indian Soc. Earthq. Technol., 39, n. 3, pp. 167-93.

SANTUCCI DE MAGISTRIS F., D'ONOFRIO A., EVANGELISTA L., FOTI S., MARASCHINI M., MONACO P., AMOROSO S., TOTANI G., LANZO G., PAGLIAROLI A., MADIAI C., SIMONI G., SILVESTRI F. (2013) – *Geotechnical characterization of the Aterno valley for site response analyses*. Rivista Italiana di Geotecnica, n. 3, pp. 23-43.

SEED H.B., IDRISS I.M. (1970) – *Soil moduli and damping factors for Dynamic Response Analyses*. Tech. Rep. EERC 70-10, Earthquake Engineering Research Centre, University of California, Berkeley.

SHIBUYA S., MITACHI T., FUKUDA F., DEGOSHI T. (1995) – *Strain rate effect on shear modulus and damping of normally consolidated clay*. Geotechnical Testing Journal, n. 18, pp. 365-375.

STOKOE K.H., DARENDELI M.B., ANDRUS R.D., BROWN L.T. (1999) – *Dynamic soil properties: laboratory, field and correlation studies*. Proceedings of the 2nd International Conference on Earthquake Geotechnical Engineering, vol. III, 1999, pp. 811-845.

UZIELLI M., LACASSE S., NADIM F., PHOON K.K. (2007) – *Soil variability analysis for geotechnical practice*. Proceedings of the 2nd International Workshop on Characterisation and Engineering Properties of Natural Soils.

UZIELLI M., MAYNE P.W. (2019) – *Probabilistic assignment of effective friction angles of sands and silty sands using quantile regression*. Georisk Special Issue in Honour and Memory of Professor Tien H. Wu "Probabilistic applications in geotechnical engineering focusing on case histories", K.K. Phoon and C.H. Juang (Eds.) DOI: 10.1080/17499518.2019.1663388

UZIELLI M., ZEI M., CASSIDY M.J. (2019a) – *Probabilistic assignment of design undrained shear strength using quantile regression*. Ching J., Li D.Q., Zhang, J. (Eds.), Proceedings, 7th International Symposium on Geotechnical Safety and Risk, Research Publishing, 899 pp.

UZIELLI M., ZEI M., FACCIORUSSO J., MADIAI C. (2019b) – *Probabilistic estimation of linear and volumetric strain thresholds for Italian clays*. Proceedings of the 29th European Safety and Reliability Conference (ESREL).

VUCETIC M., DOBRY R. (1991) – *Effect of Soil Plasticity on Cyclic Response*. Journal of the Geoechnical Engineering Division ASCE, n. 117, pp. 89-107.

VUCETIC M., LANZO G., DOROUDIAN M. (1998) – *Damping at Small Strains in Cyclic Simple Shear Test*. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 124, n. 7, pp. 585-594.

YOKOTA K., IMAI T., KONNO M. (1981) – *Dynamic Deformation Characteristics of Soil Determined by Laboratory Tests*. OYO Technical Report, n. 3.

YU K., LU Z., STANDER J. (2003) – *Quantile regression: applications and current research areas*. The Statistician, 52, n. 3, pp. 331-350.

ZHANG J., ANDRUS R.D., JUANG C.H. (2005) – *Normalized Shear Modulus and Material Damping Ratio Relationships*. Journal of Geotechnical and Geoenhnical and Geoenvironmental Engineering ASCE, 131, pp. 453-464.

Caratterizzazione probabilistica della dipendenza del modulo di taglio e del rapporto di smorzamento dalla deformazione di taglio per argille italiane

Sommario

La valutazione quantitativa della dipendenza dal modulo di taglio e del rapporto di smorzamento dal livello di deformazione di taglio costituisce un elemento fondamentale nel processo di caratterizzazione del comportamento dei terreni in campo dinamico e ciclico. Tale dipendenza può essere investigata direttamente mediante lo svolgimento di prove dinamiche e cicliche di laboratorio su campioni indisturbati, e il successivo adattamento di modelli analitici ai dati sperimentali. Tale approccio è difficilmente adottabile nell'ambito della pratica professionale a causa dell'alto costo e della durata temporale delle suddette prove, e si rende quindi conveniente fare riferimento ad approcci empirici che consentono di stimare le proprietà dinamiche dei terreni a partire da altre proprietà geotecniche che condizionano il comportamento dinamico e ciclico e che siano facilmente ottenibili quali l'indice di plasticità.

Analogamente a quanto osservato per molte altre discipline tecniche e ingegneristiche, la geotecnica sta sperimentando una costante evoluzione dal tradizionale approccio deterministico a un approccio non deterministico, nel quale la variabilità intrinseca nei dataset, le incertezze nelle misure in sito e di laboratorio, e le incertezze nei modelli utilizzati per ottenere parametri di progetto dalle misure stesse, sono modellate processate e riportate esplicitamente, con l'ausilio di tecniche quali quelle statistiche e probabilistiche. Un corpus crescente di letteratura scientifica e di applicazioni progettuali supporta l'asserzione circa l'adeguatezza dell'approccio non deterministico nel facilitare una più completa comprensione e modellazione ingegneristica della complessità dei sistemi geotecnici, e l'ottimizzazione del rapporto costi-prestazioni in fase di progetto e di realizzazione.

Questo articolo presenta un metodo empirico innovativo per la caratterizzazione probabilistica della dipendenza del modulo di taglio e del rapporto di smorzamento dalla deformazione di taglio per terreni argillosi. Il metodo consta di semplici modelli analitici, richiedenti unicamente la conoscenza anche approssimata dell'indice di plasticità e applicabili a un ampio intervallo di valori di probabilità di superamento, derivati da un database di risultati di prove di colonna risonante e taglio torsionale ciclico su 170 campioni indisturbati di argille da siti italiani. L'articolo riporta anche un esempio di applicazione del metodo a un'analisi semplificata di risposta sismica monodimensionale.