Viscoelastic analysis of one dimensional soil consolidation

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SUMMARY: The primary and secondary parts of the observed compression behaviour of soils can be separated with the help of a numerical procedure utilising digital computers, based on a chain of Kelvin units and a non linear dashpot element to represent the one-dimensional compression behaviour of soils. Distribution functions for the retardation spectrum facilitate easy computation of the viscoelastic parameters involved.

The analysis is simple, fairly accurate and compares more favourably with the experimental results than some of the typical analyses proposed earlier.

1. Introduction.

For many cases that arise in practice, Terzaghi's theory of one dimensional soil consolidation predicts the ultimate settlement fairly accurately while its estimate of the time-rate of settlement is not entirely satisfactory. The principal source of deviation from the teory is, however, what has been generally termed as secondary compression. It is now established that, though this phenomenon is significant for most of the soils after the completion of the conventional primary consolidation (the time dependent volume reduction related entirely to the dissipation of pore pressure, as was assumed by Terzaghi), the deformation due to secondary compression occurs throughout the process of consolidation.

The relative magnitudes of primary and secondary compression vary with the soil type and test conditions. The characteristic nature of the consolidation curves has been found, however, to conform to a few geometric shapes [WAHLS, 1962]. For soils which display primary consolidation predominantly, a Type I curve results as shown in Fig. 1. For soils in which the secondary compression overshadows the primary consolidation, a Type II curve as given in Fig. 1 is obtained.

The problem of actual consolidation as opposed to the Terzaghi's theoretical solution can be tackled through:

1) refinements in the Terzaghi's theoretical development of the consolidation equation

2) introduction of correlation constants between the observed and predicted behaviour

3) introduction of rheologic models to describe the stress-strain-time relationship.

SKEMPTON and BJERRUM [1957], SCHIFFMAN [1960], SCHIFFMAN and GIBSON [1964], LAMBE [1964] and CRAWFORD [1964] have all proposed methods which fall under group one given above. ROWE [1959] has studied the correlation between uniaxial and triaxial behaviour of soils under stress. But still, the recent studies have shown that the use of rheologic models is a promising line of approach for describing the consolidation phenomenon mathematically. One advantage with the use of rheologic models is that the phenomenon of secondary compression can be easily taken into account. A number of investigators [GIBSON, LO, 1961; LO, 1961; MURAYAMA, SHIBATA, 1959; SCHIFFMAN, LADD, CHEN, 1964; TAN, 1964; WAHLS, 1962; WU, RESENDIZ, NEUKIRCHNER, 1966] have already used rheologic models for the solutions of the consolidation problem.

From a review of the literature on the theories for soil consolidation, the following observations are made:

1) The chief source of deviation from the Terzaghi analysis of one dimensional soil compression is the occurrence of secondary compression. It is generally understood that both the processes of primary and secondary compression start from the instant the consolidation process commences.

2) The coefficient of consolidation (Terzaghi) is insufficient for expressing the time-rate of consolidation.

3) The secondary compression-log time relation is linear under a constant effective stress and slight deviations, if any, from this relation are not important for most of the engineering analyses.

4) Depending upon the relative proportions of primary and secondary components Type I or Type II time-compression curve as shown in Fig. 1 results.

5) Secondary compressions are mainly viscous in nature. The pore pressures set up associated with this compression are negligibly small in all soils of Type I behaviour. For Type II soils significant pore pressures may be present during the linear

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ε —log t relationship and the essentially complete dissipation of pore pressure may not coincide with the commencement of linear ε —log t relationship.

6) The dissipation of pore pressure and the

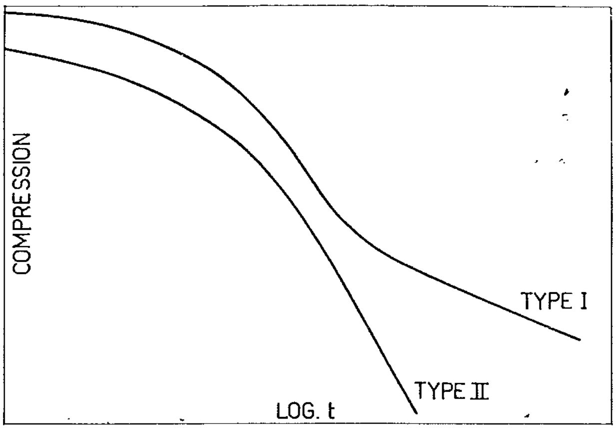

Fig. 1. - Types of consolidation curves [WAHLS, 1962].

consequent deformation can be ideally represented by a single Kelvin unit. However, to quantitatively express the phenomenon in an accurate fashion, a chain of Kelvin units or a non-linear Kelvin element is necessary.

7) The secondary compression being due to the viscous yielding of the grain structure, a dashpot is ideal to represent it. To quantitatively estimate this compression a chain of elements may be necessary or a non-linear element may be used. The models to be used should be against the effective stresses on the soil skeleton.

8) Computation through viscoelastic analysis becomes more and more involved with more finite elements used in a model or non-linearity of even a simple model. The individual parameters cannot be computed without resorting to complicated analysis or trial and error methods. The use of finite number of elements is also ambiguous and empirical.

9) In a given pressure increment, the effective stress-deformation relation can be considered to be linear without appreciable error.

10) Ultimate strain or deformation is needed for most of the published viscoelastic analyses. This involves a long duration consolidation test and is practically impossible to obtain in the case of Type II soils.

11) The first step in a complete understanding of the compressibility phenomenon and extrapolation to field behaviour, however, lies in differentiating the observed deformation at any time into its two distinct components of primary consolidation and secondary compression.

2. Proposed analysis.

It has been customary in other fields of viscoelasticity to separate the retarded elastic and viscous effects due to an applied effective stress as shown in Fig. 2, wherein the viscous effects are linear with simple time function or logarithm of time. In the case of one dimensional compression, no such demarcation of the observed behaviour is easily feasible as the flow deformation during primary compression is a function of the effective stress, itself a variable with time not easily defined. Thus a simple demarcation between the two components is not possible.

A numerical procedure is therefore proposed to obtain the primary and secondary curves from an observed experimental curve based on the following assumptions:

1) The pressure increment is suddenly applied and the total stress at every point is immediately increased by the magnitude of the pressure increment.

2) The deformation at any time 't' is small compared with the height of the sample.

3) The deformation or strain of the sample at any time 't' is the sum of primary and secondary parts.

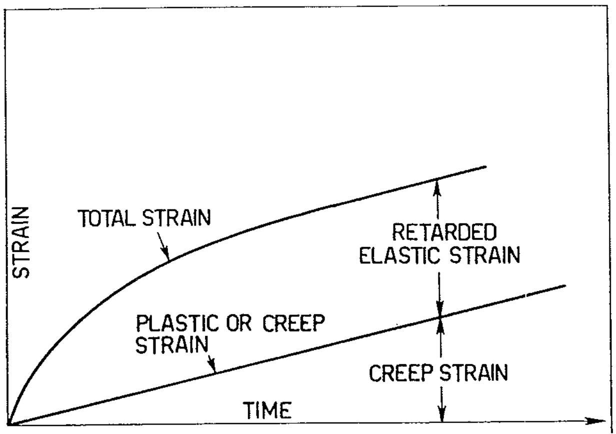

Fig. 2. - Separation of elastic and creep strains.

4) Primary and secondary phenomena commence from the instant the pressure increment has been applied.

5) The compression of the grain structure due to the primary phenomenon is linear under a given pressure increment.

6) The average effective stress on the grain structure is, therefore, a function of the primary compression at the considered time and the ultimate primary compression.

7) Secondary compressions are due to the effective stress on the grain structure.

8) Secondary compressions are viscous in nature and therefore, can be represented by a Newtonian element.

9) The viscoelastic law governing the secondary compression remains the same throughout the consolidation process under any given pressure increment.

10) The nature of secondary compression when the applied stress becomes fully effective is known and can be represented by a suitable viscoelastic expression. The ε –log time relation in the range of wholly secondary consolidation for most of the soils encountered in practice being linear, this relation has been utilised in the present analysis.

Assumptions 2 to 4 and 7,8 and 10 are all well accepted by all the investigators. Assumptions 1 and 5 are the basic assumptions of Terzaghi's theory and have been the basis for most of the analyses till now. Assumptions 6 and 9 are the logical extensions of the other assumptions.

The total strain $\varepsilon(t)$ at any time 't' under a pressure increment σ_0 is,

$$\varepsilon(t) = \varepsilon_{pr} + \varepsilon_{f1} \tag{1}$$

where ε_{pr} = strain due to primary effects or hydrodynamic effects;

ε_{f1} = strain due to secondary effects.

Using a non-linear Newtonian element with the dashpot constant changing as a function of time as,

$$\eta = \eta_s \cdot t \tag{2}$$

where η_s = a constant, η = coefficient of viscosity, the strain of the element under an effective stress $\bar{\sigma}$ is,

$$\varepsilon_{f1} = \frac{1}{\eta_s} \int_0^t \bar{\sigma} \cdot \frac{1}{t} \cdot dt \tag{3}$$

and has an asymptotic solution in the wholly secondary consolidation range as,

$$\varepsilon_{f1} = \frac{1}{\eta_s} \cdot \sigma_0 \cdot [\ln t]_{t_1}^{\infty} \tag{4}$$

where t_1 = the time necessary for the essential completion of the primary consolidation, and provides with the well known linear $\varepsilon - \log t$ relationship in the fully secondary consolidation range. The average effective stress on the soil skeleton at any time 't' is now expressed as

$$\bar{\sigma}(t) = \sigma_0 \cdot \frac{\bar{\varepsilon}_{pr}}{\varepsilon_{pr\,\infty}} \tag{5}$$

where $\varepsilon_{pr\,\infty}$ = ultimate strain due to primary consolidation.

Hence $$\varepsilon_{f1}(t) = \frac{\sigma_0}{\eta_s \cdot \varepsilon_{pr\,\infty}} \int_0^t \varepsilon_{pr} \cdot \frac{1}{t} \cdot dt \tag{6}$$

and

$$\varepsilon_{pr}(t) = \varepsilon(t) - \frac{\sigma_0}{\eta_s\ \varepsilon_{pr\,\infty}} [\varepsilon_{pr} \ln t - \sum_0^t \varepsilon_{pr} \ln t\ \Delta t] \tag{7}$$

From the experimental curve $\varepsilon(t)$, η_s, σ_0 and t are all known. As a first approximation, ε_{pr} values are assumed to be the same as the observed strains and $\varepsilon_{pr\,\infty}$ is obtained from inspection of the curve and using Eq. 7, $\varepsilon_{pr}(t)$ can be computed. This improved value $\varepsilon_{pr}(t)$ along with the improved value of $\varepsilon_{pr\,\infty}$ are substituted back in Eq. 7 to get a further refined value of $\varepsilon_{pr}(t)$. Thus after a suitable number of iterations appropriate values of $\varepsilon_{pr}(t)$ and $\varepsilon_{f1}(t)$ can be established. This process of numerical computation can be programmed for the digital computers and the experimental strain-time curve can be split into its primary and secondary parts.

From the review of the theory of viscoelasticity and one dimensional consolidation behaviour of soils, its is seen that a series of Kelvin units is ideally suited for depicting the primary effect. The strain-time relation for a chain of Kelvin units is,

$$\varepsilon(t) = J(t) \cdot \sigma_0 =$$

$$= \sigma_0 \cdot \int_0^{\infty} J(\tau) \left[1 - \exp\left(-\frac{t}{\tau_i}\right)\right] d\tau \tag{9}$$

where $J(t)$ = elastic compliance at time t;

τ_i = retardation of 'i' th Kelvin unit in the infinite Kelvin series.

It can be shown that $J(\tau) - \tau$ relation is identical with $H(\tau) - \ln \tau$ relation. Hence,

$$\varepsilon(t) = \sigma_0 \int_{-\infty}^{\infty} \bar{H}(\tau) \left[1 - \exp\left(-\frac{t}{\tau_i}\right)\right] d \ln \tau = \tag{10}$$

$$= \sigma_o \int_{-\infty}^{\infty} \overline{H}(\tau)\ d\ln\tau - \sigma_o \int_{-\infty}^{\infty} \overline{H}(\tau)\exp\left(-\frac{t}{\tau}\right) d\ln\tau \quad (11)$$

where $\overline{H}(\tau) = H(\ln\tau)$.

$$\text{But } \sigma_o \int_{-\infty}^{\infty} \overline{H}(\tau)\ d\ln\tau = \varepsilon_{pr} \quad (12)$$

Hence:

$$J(t) = \frac{\varepsilon_{pr\,\infty}}{\sigma_o} - \int_{-\infty}^{\infty} \overline{H}(\tau)\exp\left(-\frac{t}{\tau}\right) d\ln\tau \quad (13)$$

The above equation can be solved through any one of the following methods:

1) Using Laplace transform

2) Deriving a relaxation spectrum from the observed curves through approximate methods as suggested by Alfrey, Ferry and William

3) Using distribution functions for the relaxation spectrum.

Use of distribution functions has a definite advantage over the first two in the matter of simplicity of solutions obtained. The box function

$$\begin{aligned} H(\tau) &= J_o,\ \tau_3 < \tau < \tau_{max} \\ H(\tau) &= O,\ \tau < \tau_3\ ;\ \tau > \tau_{max} \end{aligned} \quad (14)$$

has been used in this analysis. Using the above

$$J(t) = \frac{\varepsilon_{pr\,\infty}}{\sigma_o} - J_o \int_{\tau_3}^{\tau_m} \exp\left(-\frac{t}{\tau}\right) d\ln\tau$$

$$= \frac{\varepsilon_{pr\,\infty}}{\sigma_o} - J_o\left[E_i\left(-\frac{t}{\tau_3}\right) - E_i\left(-\frac{t}{\tau_m}\right)\right] \quad (15)$$

where J_o = plateau compliance of the box function,

E_i = the exponential integral function,

τ_3 = minimum retardation time of the spectrum,

τ_m = maximum retardation time of the spectrum.

Tobolosky [1960] has shown the following important features of the exponential integral function:

1) If τ_A and τ_B are the intersections as shown in Fig. 3 for an experimental relaxation curve, they are related to τ_3 and τ_m as,

$$\frac{\tau_3}{\tau_A} = \frac{\tau_m}{\tau_B} = 1.781 \quad (16)$$

2) The height of the box = 2.303 J_o (17)

3) Slope of the middle straight line portion is given by,

$$-\frac{d\,\varepsilon(t)}{d\log t} = 2.303\ J_o \quad (18)$$

Thus utilising the primary compression curve in the form of $\varepsilon_{pr}(t)$ Vs log t; J_o, τ_3 and τ_m can be determined. The strain due to primary consolidation at any time 't' is then given by,

$$\varepsilon(t) = \varepsilon_\infty - \sigma_o \cdot J_o\left[E_i\left(-\frac{t}{\tau_3}\right) - E_i\left(-\frac{t}{\tau_m}\right)\right] \quad (19)$$

The strain due to secondary compression during the initial stages of consolidation process is to be obtained through the numerical integration of Eq. 6 and in the range where the applied stress has become fully effective can be computed through Eq. 4. A complete solution is thus possible to obtain

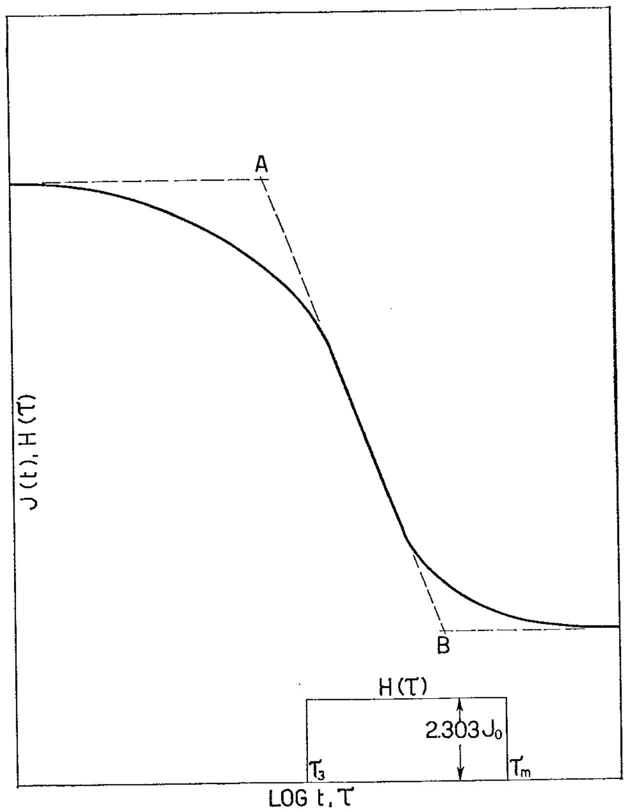

Fig. 3. - Determination of points A and B [Tobolosky, 1960].

the strain-time relation under a give pressure increment.

The above analysis has been substantiated on the basis of a number of consolidation tests carried out on a variety of soil samples obtainable and in addition, a few test results of other investigators were analysed through the proposed method. To compare the suggested method with those already proposed by some of the earlier investigators, most of the results were analysed through the models suggested by them. Detailed results of all these tests and the comparative charts are reported elsewhere [CHAR, 1969]. A typical experimental result and example of numerical computation is given below.

3. Example.

From the observed time-dial reading curve given in Fig. 4, the factor η_s is first computed. Selecting any two times t_1, and t_2 in the wholly secondary consolidation range,

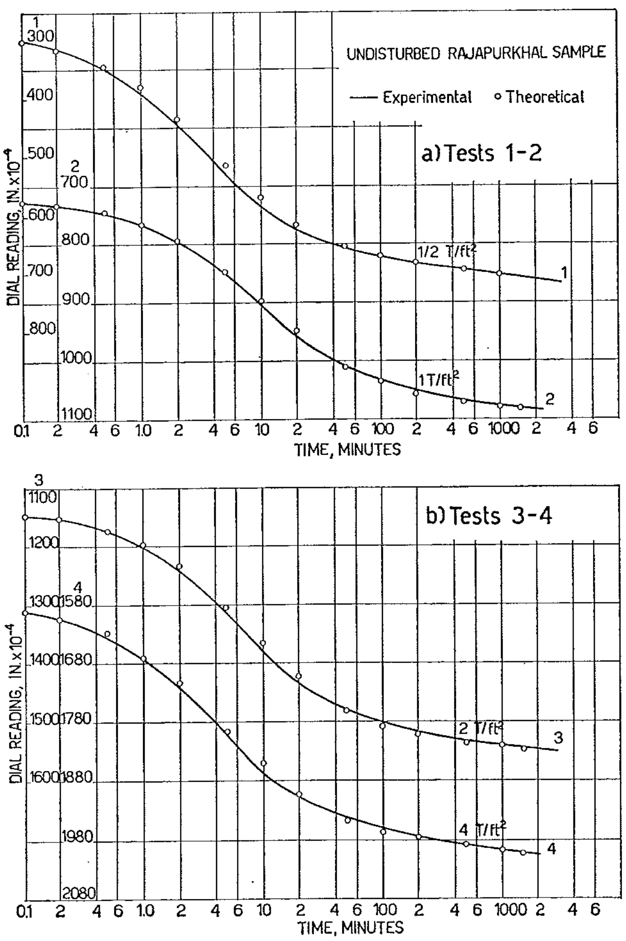

Fig. 4. - Experimental and theoretical time-compression curves.

$$\text{at } t_1 = 800 \text{ mts} \qquad \delta = 699.0$$
$$t_2 = 2000 \text{ mts} \qquad \delta = 710{,}5$$

where δ = the dial reading.

$$\text{Hence } \Delta\varepsilon = (710.5 - 699.0) \times 4/3 \times 10^{-4} = 15.3 \times 10^{-4}$$

$$\text{Using Eq. 4, } \eta_s = \frac{0.5}{15.3 \times 10^{-4}} \cdot \ln 2.5 = 0.02995 \times 10^{-4}$$

The dial reading corresponding to zero time, δ_o, can be obtained through Casagrande construction and is found to be 265. Therefore the deformations due to consolidation at all other times can be found. To complete the data for the computer, a trial value of $\varepsilon_{pr}\,\infty$ is needed. From an inspection of the curve, primary consolidation appears to be over before 50 mts. Hence the reading corresponding to this time is taken as the trial value.

$$\varepsilon_{pr}\,\infty \text{ (1st trial)} = (693.0 - 265.0) \times 10^{-4} \times 4/3 = 570.7 \times 10^{-4}$$

Strain factor, applied stress increment, η_s, $\varepsilon_{pr}\,\infty$ time 't' and the corresponding deformation - are the data to be fed into the computer using the programme given in appendix. The punched out result is given in the form of a graph in Fig. 5.

From the primary strain-time curve given in Fig. 5, $\tau_B = 16.8$. Hence $\tau_m = 1.781 \times 16.8 = 29.9$ mts.

$$\text{Now } \frac{-\,d\,J\,(t)}{d \log t} = \frac{-\,293 \times 10^{-4}}{0.5} = 2.303\ J_o$$

$J_o = 253 \times 10^{-4}$. At $t = 0.2$ mts, $\varepsilon = 66.0 \times 10^{-4}$

$$\text{From Eq. 19,} 66.0 \times 10^{-4} = 480 \times 10^{-4} - 253.0 \times 10^{-4} \times 0{,}5 \times \left[E_i\left(-\frac{0.2}{\tau_3}\right) - E_i\left(-\frac{0.2}{29.9}\right) \right]$$

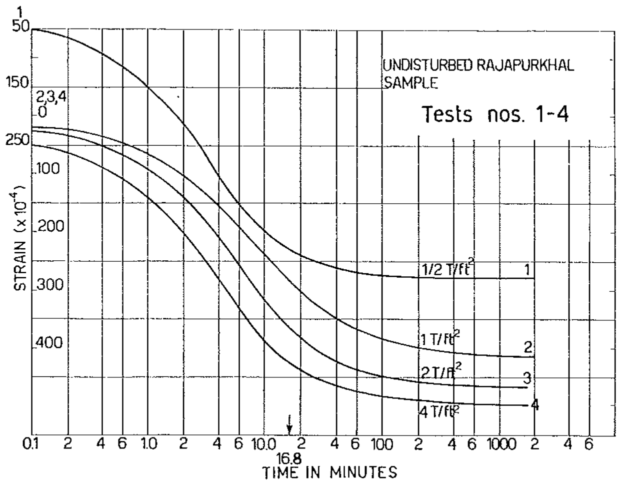

Fig. 5. - Time-primary strain curves.

Referring to the exponential integral table (20), $\tau_B = 0.936$ mts.

Using Eq. 19 primary strain at any other times can be computed as J_o, τ_3 and τ_m are all known. The strain due to secondary compression can be obtained through the numerical integration of Eq. 6. This has already been accomplished in the computer solution. These two solutions together predict the totale strain corresponding to any time 't'. The analytical solutions in the form of dial reading-time curves have been compared with the experimentally obtained results in Figs. 4 and 6.

In view of the comparisons made, it can be concluded that the analysis putforth gives an improved prediction. The deviations of any consequence occur only at the transitions. In contrast to the difficulty involved in computing the required parameters through the other methods, the proposed analysis is simple, more straightforward and the parameters can easily be determined. One advantage of the analysis is in that the ultimate total compression under a given pressure increment is not necessary, thereby a very long duration test is not required. c_v is not necessary for computation purposes as in the case of some of the other visco-

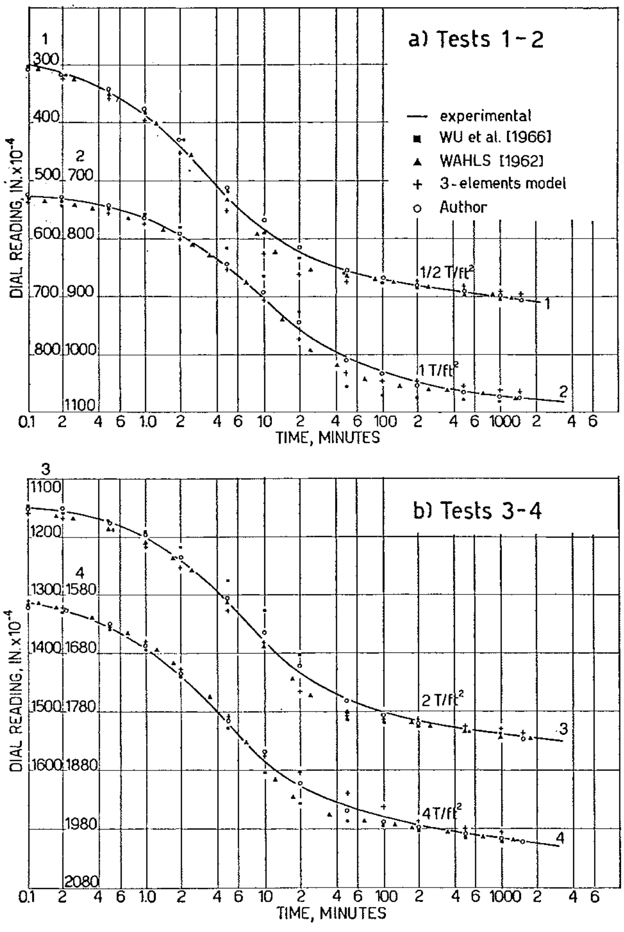

Fig. 6. - Experimental and theoretical time-compression curves.

elastic methods and hence can be used for both Type I and II soils.

The analysis has certain limitations necessarily in view of its simplicity. The first is the inability to theoretically obtain pore pressures at different points in the soil mass; only the average pore pressure in the whole mass can be obtained. This can be overcome by adopting the model at the structural level. The assumption of linear $\varepsilon_{pr} - \sigma$ relation in the analysis is fairly true at stress levels below the preconsolidation stress. The linear $\varepsilon - \log \sigma$ relation must have been used for stresses larger than the preconsolidation stress. However, as the error associated with the theoretical predictions is smaller than about 10%, refinement in this direction may not be worthwhile. In fact, Davis and Raymond [1965] observed that the linear $\varepsilon - \log \sigma$ assumption gave a solution almost similar to Terzaghi solution based on a linear $\varepsilon - \sigma$ relation.

4. Conclusions.

1) The primary and secondary parts of the observed compression behaviour of soils can be separated with the help of a numerical procedure utilising digital computers, based on an appropriate assumption as to the nature of secondary compression behaviour.

2) A chain of Kelvin units and a non-linear dashpot element together can adequately represent the one dimensional compression behaviour of soils.

3) Distribution functions for the retardation spectrum facilitate easy computation of the viscoelastic parameters involved.

4) The analysis is simple and estimates time-settlement behaviour fairly accurately and compares more favourably with the experimental results than some of the typical analyses proposed earlier.

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REFERENCES

Alfrey, T. (1948) - *Mechanical Behaviour of High Polymers.* Interscience Publishers.

Barden, L. (1965) - *Consolidation of clay with non-linear viscosity.* Geotechnique, vol. 15.

Char A.N.R. (1959): *A viscoelastic study of the one dimensional soil consolidation'.* Ph. D. Thesis, I.I.T., Kharagpur.

CRAWFORD, C. B. (1969) - *Interpretation of the consolidation test.* Journal ASCE Soil Mechanics Division.

DAVIS, E. H., RAYMOND G. P. (1965) - *A non-linear theory of consolidation'.* Geotechnique, vol. 15.

FERRY, J. D. (1961) - *Viscoelastic properties of polymers.* John Wiley, 1961.

GIBSON, R. E., LO, K. Y. (1961) - *A theory of consolidation for soils exhibiting secondary compression.* Acta Polytechnica Scandinavica.

LAMBE, T. W. (1964) - *Methods of estimating settlement.* Journal, ASCE, Soil Mechanics Division.

LO, K. Y. (1961) - *Secondary compression of clays.* Journal, ASCE, Soil Mechanics Division.

MURAYAMA, S., SHIBATA, T. (1959) - *On the secondary consolidation of clay.* Proceedings Japan Society of Testing Materials.

ROWE, P. W. (1959) - *Measurement of the coefficient of consolidation of lacustrine clay.* Geotechnique, Vol. 9.

SCHIFFMAN, R. L. (1960) - *Consolidation of soil under time dependent loading and varying permeability.* Proceedings, Highway Research Board, Vol. 37.

SCHIFFMAN, R. L., GIBSON, R. E. (1964) - *Consolidation of non-homogeneous clay layers.* Journal, ASCE, Soil Mechanics Division.

SCHIFFMAN, R. L., LADD, C. C., CHEN, A.T.F. (1964) - *The secondary consolidation of clay.* Symposium on Rheology and Soil Mechanics, Grenoble.

SKEMPTON, A. W., BJERRUM, L. (1957) - *A contribution to the settlement analysis of foundations on clay.* Geotechnique, Vol. 7.

TAN, T. K. (1964) - *Rheological parameters and the hardening coefficients of clays.* Symposium on Rheology and Soil Mechanics, Grenoble.

TERZAGHI, K. (1943) - *Theoretical Soil Mechanics.* John Wiley.

TOBOLOSKY, A. V. (1960) - *Properties and structures of polymers,* John Wiley.

WAHLS, H. E. (1962) - *Analysis of primary and secondary consolidation.* Journal, ASCE, Soil Mechanics Division.

WENZEL, L. K. (1942) - *Methods for determination of permeability of water bearing materials.* U.S. Geological Survey Water Supply, Paper 887.

WU, T. H., RESENDIZ, D., NEUKIRCHNER, R. J. (1966) - *Analysis of consolidation by Rate Process Theory.* Journal, ASCE, Soil Mechanics Division.

APPENDIX: COMPUTER PROGRAMME

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      DIMENSION  T(50),S(50),E(50),EB(50),EPR(50),EI(50),EDT
        (50),G(50)
   88 READ 8,K,NR
      READ 1,FAC,STR,VIS,EPF
      L=O
      DO 12 I=1,NR,5
   12 READ 2,T(I),T(I+1),T(I+2),T(I+3),T(I+4)
      DO 15 I=1,NR,5
   15 READ 2,S(I),S(I+1),S(I+2),S(I+3),S(I+4)
      DO 22 I=1,NR
   22 EI(I)=FAC*S(I)
      NS=NR—I
      EDT(1)=O.
      EB(1)=O.
      G(1)=O.
      DO 36 I=1,NR
   36 E(I)=EI(I)
   98 DO 24 I=2,NR
   24 EDT(I)=(E(I)—E(I—1))/(T(I)—T(I—1))
      FM=STR/(VIS*EPF)
      DO 62 I=2,NR
   62 G(I)=G(I—1)+EDT(I)*LOGF(0.5*(T(I)+T(I—1)))*(T(I)—
      —T(I—1))
      DO 42 I=2,NR
   42 EB(I)=FM*(E(I)*LOGF(T(I))—G(I))
      DO 48 I=1,NR
   48 E(I)=EI(I)—EB(I)
      EPF=E(NR)
      L=L+1
      IF (L—K) 98,99,99
   99 DO 84 I=1,NR
   84 EPR(I)=EI(I)—EB(I)
      PUNCH 6
      PUNCH 9
      DO 74 I=1,NR
   74 PUNCH 5,T(I),EB(I),EPR(I)
      GO TO 88
    8 FORMAT I2,I2
    1 FORMAT (4F10.0)
    2 FORMAT (5F10.0)
    5 FORMAT (3X,F7.1,10X,F10.4,10X,F10.4)
    9 FORMAT (4X,5H,TIME,14X,5HEFLOW,14X,
        8HEPRIMARY)
    6 FORMAT (25X,7HRESULTS)
      END
```


SOMMARIO

Interpretazione viscoelastica del processo di consolidazione monodimensionale.

Si osserva preliminarmente che l'andamento dei cedimenti che si verificano in un terreno di fondazione per effetto di un incremento delle sollecitazioni esterne è spesso difforme da quello previsto dalla teoria della consolidazione monodimensionale di Terzaghi, e ciò specialmente a causa dell'effetto secondario, la cui influenza, secondo le più moderne vedute, si manifesterebbe anche molto prima del termine del processo di consolidazione attribuito alla semplice espulsione dell'acqua. La deformazione nel generico istante t risulterebbe, quindi, dalla somma di una componente primaria ε_{pr} e di una secondaria ε_{fl} come nella (1).

L'importanza relativa delle due componenti della deformazione è diversa secondo il tipo di terreno. In fig. 1 sono indicate, a titolo di esempio, due curve caratteristiche, rispettivamente, di un terreno in cui predomina l'effetto primario (tipo I) e di un altro, invece, in cui l'effetto secondario è tanto cospicuo da alterare completamente la forma della curva (tipo II).

Per una migliore interpretazione delle curve cedimenti tempo che si ottengono con l'edometro si può procedere, innanzitutto, modificando la teoria originaria di Terzaghi per tenere conto di fattori che la teoria stessa trascura. Un altro metodo consiste nell'applicare la teoria introducendo dei coefficienti correttivi dei risultati sperimentali.

Infine, si è manifestata con sempre maggiore frequenza

la tendenza ad abbandonare il semplice modello reologico di Terzaghi per fare ricorso a modelli più avanzati che tengano conto adeguatamente dello effetto secondario.

Nel presente articolo, che è un estratto della tesi per il dottorato conseguito presso l'*Indian Institute of Technology* di Kharagpur, India, l'A. segue questa terza via per proporre un metodo di interpretazione delle curve sperimentali cedimenti-tempo, che si rilevano con l'edometro.

Dagli studi già effettuati da vari autori sull'argomento si deduce che il cedimento secondario si sviluppa all'incirca con legge logaritmica in funzione del tempo e che questa componente della deformazione fa risentire la sua influenza sin dall'inizio del processo. Risulta, inoltre, che per terreni con comportamento del tipo II di fig. 1 le pressioni neutre possono essere ancora apprezzabili anche nel tratto in cui la curva ε, lgt ha un andamento all'incirca lineare.

È noto, infine, che dal punto di vista reologico la deformazione conseguente all'espulsione dell'acqua può rappresentarsi con un elemento non lineare di Kelvin ovvero con una serie di elementi di Kelvin, ciascuno dei quali costituito da un elemento di Newton in parallelo con un elemento di Hooke. Il processo secondario, di tipo viscoso, può, invece, rappresentarsi con un elemento non lineare di Newton ovvero con una serie di elementi di Newton, facendo sempre riferimento alle pressioni effettive.

L'A. propone che la deformazione complessivamente raggiunta dal provino all'istante generico t sia rappresentata dalla (1), nell'ipotesi che i processi che danno luogo alle due componenti primaria e secondaria della deformazione abbiano avuto entrambi inizio nell'istante $t = 0$ in cui è stata applicata la pressione esterna σ_o.

Se si accetta una delle ipotesi fondamentali della teoria della consolidazione di Terzaghi, la componente primaria della deformazione dipende linearmente dalla pressione effettiva; ne segue che la pressione effettiva $\sigma(t)$ mediamente raggiunta nel provino al tempo t può esprimersi con la (5) in funzione del rapporto fra la deformazione primaria ε_{pr} raggiunta al tempo t ed il valore finale $\varepsilon_{pr}\infty$ della deformazione medesima.

D'altra parte, la componente secondaria ε_{f1} della deformazione può esprimersi facendo riferimento ad un elemento di Newton non lineare con la formula (3) in cui η_s è una costante di viscosità.

Per valori di $t > t_1$, dove t_1 è il tempo occorrente per completare convenzionalmente la consolidazione primaria, la (3) assume il noto andamento rettilineo fornito dalla (4) che si deduce dalla (3) ponendo $\overline{\sigma} = \sigma_o = \text{cost}$. Introducendo, allora, nella (3) l'espressione (5) della $\overline{\sigma}(t)$ si deduce la (6) che esprime la deformazione secondaria in funzione di quella primaria e del tempo e nella quale compaiono le costanti σ_o, η_s e $\varepsilon_{pr}\infty$. La σ_o è un dato del problema; la η_s può ricavarsi dalla curva sperimentale $\varepsilon(t)$ applicando la (4) al tratto finale rettilineo; il valore $\varepsilon_{pr}\infty$ può dedursi per tentativi dalla formula (7) con un programma di calcolo riportato in appendice, assumendo nel primo tentativo $\varepsilon_{pr}(t) = \varepsilon(t)$).

Esempi delle curve $\varepsilon_{pr}(t)$ che si deducono dalla (7) con il procedimento esposto, sono riportati in fig. 5. Queste curve possono interpretarsi con un modello reologico costituito di una serie di elementi di Kelvin, utilizzando la formula (19), in cui i simboli hanno il significato definito dalla (14) e seguenti.

In definitiva la (19) fornisce l'andamento nel tempo della componente primaria della deformazione e la (6) della componente secondaria: è possibile così analizzare i risultati sperimentali separando queste due componenti della deformazione.

Nelle figg. 4 e 6 l'A. mostra le curve che con il procedimento indicato interpretano i risultati sperimentali di prove di compressione edometrica effettuate dall'A. medesimo e da altri ricercatori per diversi valori della pressione applicata σ_o. Le esperienze trovano in tali curve una soddisfacente interpretazione.

Si osservi ancora che per l'applicazione del metodo non è necessario effettuare prove di lunga durata, bastando all'uopo riconoscere un tratto finale rettilineo sufficientemente esteso della curva cedimenti tempo nel diagramma semilogaritmico. Naturalmente, non è possibile valutare le pressioni effettive e neutre in ogni punto del terreno, così come con la teoria di Terzaghi, ma solo i valori medi che assumono queste grandezze nell'istante generico, relativamente all'intero volume di materiale.