Dynamic stability approach for shield driven tunnels

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SUMMARY: The influence exerted by the time factor on the ground deformation regime in soft ground tunnelling is well known.
This factor can be expressed as a function of the rate of tunnel advance and the rate of deformation of the soil adjacent to the opening.
For shield driven tunnels, the problem lies in the accommodation of the time factor into the ground loss computations.
In the present paper an effort has been made towards deriving some qualitative relationships which relate the time factor with stability factors, tunnel and shield geometry, and finally the total volume of ground loss due to tunnel excavation.

1. Introduction

Excavation of a shield-driven tunnel in soft ground results in a considerable change of the state of stress in the surrounding soil. This change is associated with ground movements which tend to invade the tunnel face and fill the space behind it.

Ground disturbance is reflected upwards in the surface above the tunnel by the creation of the well know rather symmetrical settlement trough. Loss of ground around and due to the exacavation is related to the volume included in the surface subsidence curve per unit length of tunnel advance.

It is accepted that ground loss is a function of tunnel and shield geometry, soil properties, any stabilising factor applied i.e.: the action of compressed air, and the position of the groundwater level.

With very few exceptions ground stability in soft ground tunnelling has been treated — so far — as a static concept with the aid of semi-empirical failure criteria like the simple overload factor (OFS).

Yield conditions for the soil surrounding unsupported tunnels are associated with the creation of a « plastic annulus » around the opening. Theory of elasticity defines the radius of the plastic zone and the parameters influencing its amplitude. In the present paper using Tresca's criterion of failure a function was given relating the magnitude of the radius of plastic zone (R) to the stability criterion having the form of the simple overload factor (OFS).

The paper also deals with the employement of the « time factor » to stability considerations. The analysis points out all major sources of ground loss during excavation. As a conclusion an original qualitative relationship is proposed relating the total volume of ground loss to the rate:

I of tunnel advance and

II of clay deformation in the vicinity of the tunnel.

Finally, although this approach has — of course — its limitations especially in the definition of the rate of clay deformation it is believed that provides ground towards further investigations of dynamic stability analysis in shield-driven soft ground tunnelling.

2. Static stability criteria

DEERE *et al.* [1969] proposed that the stability and the potential ground loss for a tunnel in clay might be expressed as a function of a « simple overload factor », OFS, which is the ratio of the overburden pressure, less any internal pressure (for instance, air pressure if it is applied), to the undrained shear strength of the clay for conditions in which the vertical and lateral pressures pre-existing in the ground are equal.

Thus,

$$OFS = \frac{\sigma_v - \sigma_i}{c_u} \qquad (1)$$

The maximum tangential (hoop) stress according to the theory of elasticity equals twice the radial (vertical) pressure σ_v, for $K = 1$.

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Thus, one may define the « modified overload factor », OFM, as

$$OFM = \frac{\sigma_{\theta\,max} - 2\sigma_i}{q_u} \qquad (2)$$

In essence, the maximum tangential stress is the major principal stress at the tunnel wall surface, and it is reasonable to assume that when σ_θ exceeds q_u some shearing take place to form a plastic annulus around the unsupported tunnel. The radius of the sheared annulus depends upon the magnitude of the ratio σ_θ / q_u.

In Soil Mechanics, considering the case of a frictionless soil under $K_o = 1$ conditions, it is known that the yield state must satisfy TRESCA's criterion of failure,

$$\sigma_1 - \sigma_3 = 2\,c_u$$

This could be written as:

$$\frac{\sigma_1 - \sigma_3}{2\,c_u} = a \qquad (3)$$

where $a = 1$ at equilibrium.

Substituting $\sigma_1 = \sigma_v$ and $\sigma_3 = \sigma_i$ then,

$$a = \frac{\sigma_v - \sigma_i}{2\,c_u} = \frac{OFS}{2} \qquad (4)$$

$a < 1$ means that no plastic zone will develop whereas

$a > 1$ means that a plastic zone will develop.

The radius of a developed plastic zone is given by equation (5). Thus:

$$R = R_o \exp\left[\frac{\sigma_v - \sigma_i}{2\,c_u} - \frac{1}{2}\right]$$

or

$$R = R_o\, e^{0 \cdot 5\,(OFS - 1)} \qquad (5)$$

This relationship has been plotted on a log-linear graph (see Figure 1) for values of R_o ranging from 1 m to 4.5 m. Although the graph is self-explanatory, it must be emphasised that for the critical values of OFS = 6.28 [as defined by DEERE *et al.* 1969] the extent of plastic zone is contained between the limits 14.5 m < R < 68 m depending upon the respective value of the tunnel radius R_o. It should be noted, however, that for basic stability, OFS should not exceed 6 [PECK, 1969].

A detailed presentation of ten case studies has been given by PECK in his State of the Art Report. It was concluded that tunnelling can be carried out without undue difficulties in plastic clays if OFS ⩽ 5. In shield tunnelling, if OFS is much greater than this, the clay is likely to invade the tail-piece too rapidly to permit satisfactory filling of the void with pea gravel or grout. For OFS values approaching 7, the shield may become unmanageable because of its tendency to tilt as it advances.

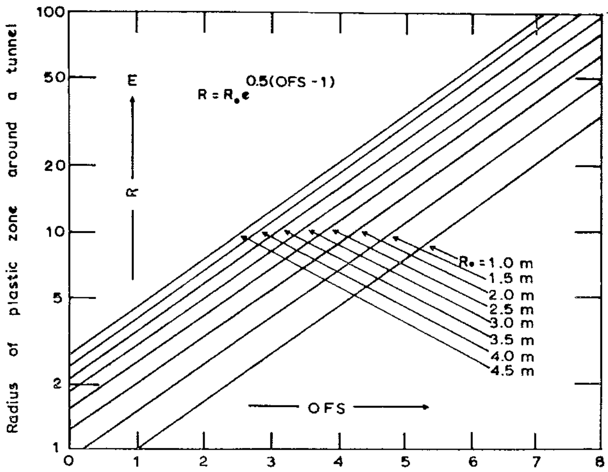

Figure 1. - Relationship between the extent of a plastic zone around a tunnel (R) and the OFS for various tunnel radii (R_o).

Using PECK's *op cit* published data a graph has been ploted relating the dimensionless ratio: tunnel depth (Z) tunnel diameter (D) i.e. Z/D; to OFS for ten well documented case studies of tunnelling.

As is shown in Figure 2 a curve of the second degree describes the increase of OFS as Z/D decreases. Taking into account the fact that shear strength may reasonably be assumed to increase linearly with depth the critical value of OFS for shallow depths could be even less than 6. As MUIR WOOD [1970] pointed out, this is only one condition for stability. DEERE *et al.* [1969] emphasized the importance of the time of exposure of the face in a soil, the effective permeability of which is sufficiently high to permit appreciable variation in pore water distribution during the period of exposure. Immediately after excavation, the release of the ground sets-up negative pore pressures at the tunnel face which provide some measure of soil self-support so long as the condition persists and this negative pressure is assisted by the cohesion of the soil. Although ground movements and strain associated with soft ground tunnelling have been studied to some detail

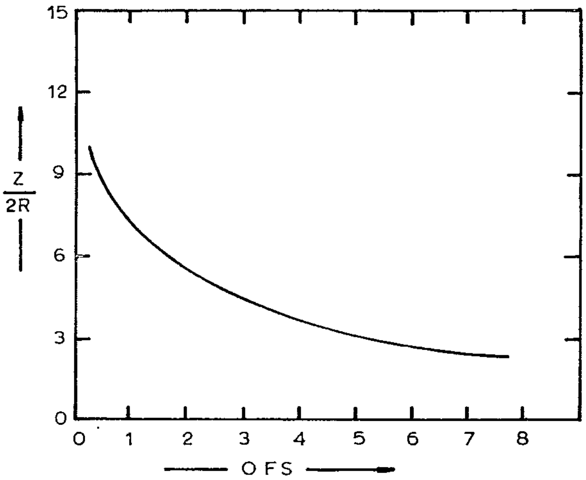

Figure 2. - Relationship between the ratio Z/2R and OFS. Data are taken from PECK (1969) p. 229.

for the area adjacent to the tunnel face [MYRIANTHIS, 1975], there are limited information for the stress changes in the same area. It is believed that employment of the stress-path method and laboratory simulation of ground conditions prevailing in the tunnel face is probably the best approach for tackling the stress problem.

3. **Surface settlement and loss of ground**

The excavation of a tunnel in clay, under normal constructional and ground conditions, creates a symmetrical settlement trough at the ground surface. The shape of the trough is nearly independent of the magnitude of the maximum settlement and the settlement volume is equal to the volume of lost ground in the tunnel modified by any volume change in the subsiding mass.

More recently, PECK *et al.* [1972] have stated that the maximum amplitude of the settlement curve can be estimated on the assumption that the volume of the settlement trough will be about one per cent of the volume of the tunnel (that is, the volume of the excavated soil). Under exceptionally good conditions and workmanship, the settlement may be as little as half of this amount. In contrast, volumes of settlement of up to 40% or 50% of the volume of the tunnel are not unknown.

The symmetrically-shaped settlement profile over a tunnel can adequately be approximated by a Gaussian error curve, and it has been shown in the literature that the shape of most settlement profiles conforms closely to it. (Figure 3). From the properties of the Gaussian error function it is known that the surface settlement volume per unit advance of the tunnel is proportional to the product of i (the

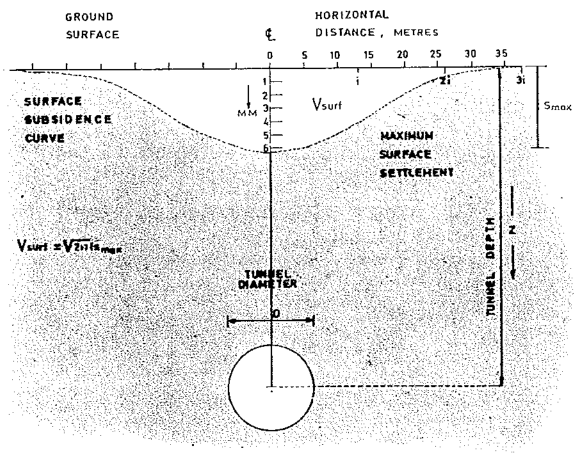

Figure 3. - Symmetrical surface settlement profile over a circular tunnel. It is accepted that the curve can adequately be approximated by a Gaussian error function.

standard deviation on a normal probability curve, being the point of inflection on the surface settlement semi-profile) and s_{max} the maximum settlement on the error curve. Thus,

$$V_{surf} = \sqrt{2\pi}\, i\, s_{max} \qquad (6)$$

Loss of ground however, must be related to the theoretical excavated volume of the tunnel (V_{exc})

$$V_{surf} = V_l\, V_{exc} \qquad (7)$$

where V_l is the ground lost.

SCHMIDT [1969] examined variations in the loss of ground with the OFS value and with the soil properties under $K = 1$ conditions on the assumption of no volume change (that is $\nu = 0.5$).

He concluded that:

$$\text{For OFS} \geq 1, \quad V_l = \frac{3\, c_u\, e^{OFS - 1}}{E} \qquad (8)$$

$$\text{For OFS} \leq 1, \quad V_l = 3\ \text{OFS}\, \frac{c_u}{E} \qquad (9)$$

Finally, he pointed out that the strength/ /modulus ratio for common soils varies within a fairly narrow range approximately bounded by the values 5×10^{-3}. This range is likely to be narrowed as more becomes known about the deformational behaviour of clay soils.

The dimensionless ratio: tunnel depth over tunnel diameter (Z/D) is a key constant related to the maximum surface settlement of the ground above the tunnel [MYRIANTHIS, 1974 and MYRIANTHIS 1974 a] and to the loss of ground (V_l). Finally, according to SCHMIDT [1969] the ratio Z/D is a function of OFS, because $V_l = f(OFS)$.

4. Dynamic stability considerations

The problem of tunnel stability is usually treated as a static or pseudo-static concept, although ground stability conditions change in every step of excavation, lining and tunnel stabilisation.

Tunnelling engineers know from practical experience that the slower the rate of tunnel advance the greater is the total clay intrusion for a given depth of tunnelling. Time factor is a governing parameter in the ground stability regime around a tunnel.

The rate of tunnel advance determines in effect the time of exposure of any element of clay at or near to the tunnel face as well as around the opening.

In soft ground tunnelling which is the case of many urban tunnels the excavation is made with shield. However, dynamic stability considerations might be faced through the accomodation of the time factor into ground loss computations taking, also, into account shield geometry. (Figure 4).

MUIR WOOD [1970], in a quite precise manner, pointed out the main contributory factors for the determination of ground loss. Using his arguments as a framework, an attempt has been made by the author to modify and extend the concept, emphasizing the role of rate effects. The total ground loss associated to tunnel excavation might be expressed as the sum of ground losses:

1. At the tunnel face
2. Behind the bead of the shield
3. Along the shield, and
4. Behind the tail of the shield.

Thus,

$$V_l = V_{l_1} + V_{l_2} + V_{l_3} + V_{l_4} \qquad (10)$$

The first factor of equation (10) can be expressed in terms of the shield radius (R_s) and the horizontal movement of ground at the

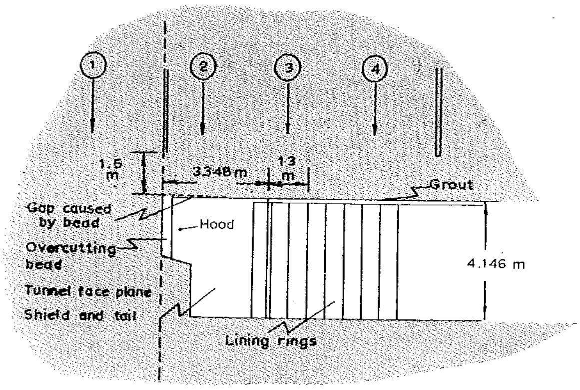

Figure 4. - Major ground loss areas around a shield-drives tunnel. Vertical settlement caused principally by:
(1) intrusion into the face,
(2) closure of the bead gap,
(3) closure of the ungrouted annulus between lining and ground,
(4) closure of the grouted annulus.
After ATTEWELL and FARMER, (1974).

face per unit length of shield's advance (j).

Thus,

$$V_{l_1} = \pi R_s^2 j \qquad (11)$$

This factor obviouslyis not time dependent. In order to incorporate the time factor it is necessary to define two basic rates, namely the rate of clay movement at the tunnel face ($\alpha = ds/dt$), and the rate of tunnel advance ($A = dl/dt$). Note that l represents length measured in metres. Therefore,

$$j = \alpha / A \qquad (12)$$

and

$$V_{l_1} = \frac{\pi R_s^2 \alpha}{A} \qquad (13)$$

Equation (13) expresses the ground loss due to the face take area.

Assuming a 180 degrees bead, the ground loss due to the radial take area is,

$$V_{l_2} = \frac{\pi l_o}{2}(2 R_s - d) \simeq \pi l_o R_s$$

because $$d << 2 R_s \tag{14}$$

On this basis one could define the « exposure time » (t_{exp}) for an element of clay above the tunnel crown as,

$$t_{exp} = \frac{l_o}{A} \tag{15}$$

During that time the displacement of the clay element is:

$$s_{exp} = \frac{l_o \alpha}{A} \quad \text{or} \quad s_{exp} = l_o j \tag{16}$$

From equations (14) and (16) the ground loss could be expressed more accurately as,

$$V_{l_2} = \pi l_o^2 R_s \frac{\alpha}{A} \quad \text{or} \quad V_{l_2} = \pi l_o^2 R_s j \tag{17}$$

Since the bead is relatively small, (something between 5 to 25 mm according to European standards), one should expect that during the shield's passage there are two distinct alternatives, i.e. either,

$s_{exp} > d$ which means that the bead is closed;

or

$s_{exp} < d$ which means that the bead is not closed.

In difficult soils the shield is often distorted so that its cross-section changes along its length. This results in some extra ground loss. The same, however, could happen when the shield is driven with its axis at an angle to the axis of the tunnel.

Muir Wood *op cit*, proposed that if a shield « crabs » or, on account of poor ground is driven at an appreciable attitude, considerable settlement and ground losses are likely to occur.

Hence,

$$V_{l_3} = \frac{\pi l_o \lambda}{8} \tag{18}$$

A substantial ground loss usually occurred behind the tailpiece before, during and after the ground stabilization through grouting. The estimation of that component of ground loss is probably the most difficult and speculative because many and different factors are affecting the nature and extent of ground movement behind the tailpiece. First of all, the soil's nature, its stiffness, cohesion and moisture content are the dominant factors. Secondly, but not less important, is the type of temporary supporting system as well as the type of lining.

The time of unsupported ground exposure, the composition and effectiveness of grouting, the tunnel's depth and its location with respect to the groundwater level are no doubt some additional factors.

As a first approximation, it could be argued that for tunnels above the groundwater level, the loss of ground is given by:

$$V_{l_4} = 2 \pi R_s (R_s - R_o) \tag{19}$$

Finally, taking into account the partial components of ground losses as they are expressed in equations (13), (15), (18) and (19) it is possible to derive a qualitative linear relationship relating total ground loss per unit length of tunnel (V_l) to the time factor [1] $(\frac{\alpha}{A})$.

Thus,

$$V_l = \frac{\alpha}{A} \pi R_s (R_s + l_o^2) +$$

$$+ 2\pi_4 (\frac{l_o \lambda}{16} + R_s - R_s^2 R_o) \tag{20}$$

This important relationship reveals that total volume of ground loss is proportional to the rate of clay deformation and inversely proportional to the rate of tunnel advance. This is already well known and supported from practical experience.

A final comment to equation (20) is that both shield and tunnel geometry (R_s, l_o, R_o) as well as excavation parameters (A, λ) are known and measurable quantities. Therefore, providing that the rate of clay deformation can be determined from special laboratory (extrusion techniques) or in-situ measurements there is a possibility for future quantitive exstimate of ground loss for shield driven tunnelling in soft ground. The latter remark eventually provides ground for further investigation.

(1) In essence this ratio as a fraction of two rates expresses the increment of clay deformation over the increment of tunnel advance per unit of time.

5. Conclusions

The stability of soft ground tunnels can be examined with the aid of some semi-empirical criteria such as those expressed by the simple overload factor (OFS) and the modified overload factor (OFM).

The loss of ground around the opening is probably a major factor contributing to ground movements and surface settlements. Static stability analysis indicates that loss of ground is a function of OFS.

The incorporation of the time factor into any soft ground tunnelling stability consideration is a real necessity because stability is a dynamic phenomenon rather than a mere static concept.

In fact, the total volume of ground loss is a function of time dependent parameters such as the rate of tunnel advance, and the rate of clay deformation around the opening. It is also function of any applied stabilising factor and the tunnel and shield geometrical elements.

The accuracy of the proposed qualitative relationship between the volume of total ground loss and time factor depends primarily upon the nature of the soil and its rheological properties and secondly upon the groundwater regime existing near the face and around the circumference. Maybe it is reasonable to suppose that the function in question is valid for short time domains, such as, for instance the time elapsed between the excavation and the installation of the early support of the tunnel.

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APPENDIX - 1: NOTATION

1.	$a = \dfrac{\sigma_1 - \sigma_3}{2\,c_u}$	
2.	$a = ds/dt$	rate of clay movement at the tunnel face
3.	$A = dl/dt$	rate of tunnel advance (l: length measured in metres)
4.	c_u	undrained shear strength of the clay
5.	d	width of the bead
6.	D	tunnel diameter for a circular opening
7.	i	the standard deviation on a normal probability curve or equally the point of inflection on the surface setlement semi-profile
8.	j	horizontal movement of ground at the tunnel face per unit length of shield's advance
9.	K	coefficent of earth pressure
10.	K_o	earth pressure at rest
11.	l_o	length of the shield
12.	λ	the « look-up » of shield measured as extent of out of plumb on vertical diameter
	π	3.14
13.	V	Poisson's ratio
14.	OFS	simple overload factor
15.	OFM	modified overload factor
16.	q_u	unconfined compressive strength of the clay
17.	R	radius of plastic zone around a tunnel
18.	R_o	radius of tunnel
19.	R_s	shield radius
20.	S_{max}	the maximum settlement measured on the Gaussian error curve
21.	S_{exp}	displacement of a clay element during exposure time
22.	σ_v	overburden pressure
23.	σ_i	internal pressure applied to the excavated tunnel wall for stabilisation purposes
24.	σ_1	major principal stress

25.	σ_3	minor principal stress
26.	$\sigma_{\theta_{max}}$	maximum tangential stress around the tunnel opening
27.	t_{exp}	exposure time for an element of clay above the tunnel soffit
28.	V_{surf}	volume of the surface settlement curve per unit advance of the tunnel
29.	V_{exc}	the theoretical volume of the tunnel per unit advance
30.	V_l	volume of the ground loss
31.	Z	depth of tunnel i.e. ground surface to the tunnel axis
32.	E	Young's modulus

SOMMARIO

Stabilità di gallerie con scudo

Lo scavo di una galleria con scudo in terreni molli provoca una sensibile variazione dello stato tensionale nel terreno circostante, nonché movimenti del terreno che tende ad invadere il fronte di scavo ed a riempire gli spazi a tergo dello scudo.

In superficie, ciò provoca la nota deformata simmetrica; il volume di cedimento per unità di lunghezza della galleria viene posto in relazione con la perdita di terreno (ground loss) dovuta allo scavo. Tale perdita di terreno è funzione della geometria della galleria e dello scudo, delle proprietà del terreno, del livello della falda e dell'eventuale uso di provvedimenti di stabilizzazione, quali aria compressa, all'interno dello scavo.

Con poche eccezioni, il problema è stato finora affrontato su basi statiche con l'aiuto di criteri empirici, quale il « fattore di sovraccarico semplice OFS » ovvero il « fattore di sovraccarico modificato OFM ».

DEERE *et al.* (1969) proposero di esprimere la stabilità e la potenziale perdita di terreno per una galleria in argilla in termini di un fattore di sovraccarico semplice, OFS, dato dal rapporto fra la pressione litostatica alla profondità dello scavo, meno l'eventuale pressione applicata all'interno dello scavo, e la coesione non drenata dell'argilla, e questo per condizioni litostatiche di tipo sferico (K = 1). Si ha cioè:

$$OFS = \frac{\sigma_v - \sigma_i}{c_u} \quad (1)$$

Secondo la teoria dell'elasticità, la massima tensione tangenziale al bordo dello scavo $\sigma_{\theta max}$ è pari a 2 σ_v per K = 1.

Può allora definirsi un fattore di sovraccarico modificato

$$OFM = \frac{\sigma_{\theta max} - 2\sigma_i}{q_u} \quad (2)$$

dove $q_u = 2\ c_u$ è la resistenza a compressione dell'argilla.

Dalla teoria dell'elasticità è noto che, intorno ad una galleria di raggio Ro, si forma una zona plastica di raggio R. Applicando i concetti della meccanica dei terreni è possibile ricavare la relazione (5), che è posta in diagramma in fig. 1. Da tale diagramma risulta che, in corrispondenza di OFS = 6.28 [valore critico secondo DEERE *et al.* 1969] il raggio R della zona plastica varia fra 14.5 e 68 m, a seconda del valore di Ro. È da notare che, secondo PECK [1969], il valore di OFS non dovrebbe superare 6 perché sia assicurata la stabilità.

Utilizzando una serie di dati empirici pubblicati da PECK [1969], è stata preparata la fig. 2, che pone in relazione OFS con il rapporto Z/D fra la profondità ed il diametro della galleria.

L'esecuzione di una galleria in argilla produce cedimenti alla superficie del terreno, con un profilo simmetrico che può essere ben approssimato da una funzione di Gauss. Detto i il parametro di tale curva, ed Smax il massimo cedimento in asse alla galleria, il volume di cedimento in superficie per unità di lunghezza della galleria vale:

$$Vsurf = \sqrt{2\pi}.\, i.\, Smax$$

Detto Vexc il volume teorico di scavo, e Ve la perdita di terreno, si ha:

$$Vsurf = Ve \cdot Vexc$$

Viene mostrato che il rapporto Z/D è funzione di Ve e, secondo SCHMIDT [1969], anche di OFS.

In effetti, SCHMIDT ricavò un abaco dei valori teorici di Ve in funzione di OFS.

Si ammette usualmente che, per una data profondità, l'ingresso di argilla nello scavo sia tanto maggiore quanto minore è la velocità di avanzamento della galleria.

In effetti, il tempo è un fattore di grande importanza per le condizioni di stabilità nel terreno circostante una galleria.

Per gallerie scavate con scudo (fig. 4), MUIR WOOD [1970] ha elencato con precisione i fattori che determinano la perdita di terreno; il problema è di introdurre nella loro valutazione l'effetto del tempo.

Un tentativo in questo senso viene suggerito, basandosi sullo schema di MUIR WOOD [1970].

La perdita totale di terreno può essere espressa (eq. 10. fig. 4) come somma delle perdite:

1. Al fronte dello scudo
2. Dietro l'imbocco dello scudo
3. Lungo lo scudo
4. A tergo della parte terminale dello scudo

Con una serie di sviluppi viene ricavata una relazione (eq. 20) fra Ve e due parametri temporali: la velocità di movimento dell'argilla al fronte di scavo $\alpha = ds/dt$ e la velocità di avanzamento della galleria (A = dl/dt).

In conclusione, viene messo in evidenza che la perdita di terreno è uno dei più importanti fattori nel determinare il cedimento in superficie; l'analisi di stabilità mostra che essa è funzione di OFS.

È necessario portare in conto l'effetto del tempo nelle considerazioni di stabilità, in quanto l'intero processo è dinamico piuttosto che statico. In effetti, la stabilità risulta funzione di parametri che dipendono dal tempo, quali α, A e la velocità di applicazione di un'eventuale pressione di stabilizzazione all'interno della galleria.