Elastic modelling of inherently anisotropic behaviour in sands

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SUMMARY: Inherent anisotropy is a characteristic of most undistrutbed sand deposits, and influences all stress-strain analyses of the soil.
An exhaustive experimental investigation carried out on laboratory-reconstructed sand samples allowed determination of the elastic parameters reproducing the anisotropic behaviour of the soil.
These values were introduced into a numerical finite-element model reproducing the behaviour of a shallow foundation and of the soil, with the aim of evaluating the inherent anisotropy effect on stress distribution and elastic settlement in the soil itself.

1. Introduction

The formation of natural in situ sand deposits is often due to sedimentation of the soil particles, which normally takes place in a fluid medium such as water or air. This gravitational process creates a typical granular soil fabric in which parallel horizontal stratification planes may be distinguished: stratification is principally due to the shape of the sand particles which show preferential orientation during deposition. In this case, the structure of the deposit may be referred to a physical model called inherent anisotropy. This anisotropy may be present before the soil is strained and must be distinguished from the other type of anisotropy, called induced. Inherent anisotropy is therefore defined by ARTHUR and MENZIES [1972] as «a physical characteristic inherent in the material and entirely independent of the applied strains», while induced anisotropy is defined as «a physical characteristic due exclusively to the strain associated with an applied stress».

The first researchers to model inherent anisotropy in granular soil were probably CASAGRANDE and CARRILLO [1944]; the subject was then studied by numerous researchers [JOHNASSON, 1965; PARKIN *et al.*, 1968; ARTHUR and DUNSTAN, 1969]. However, the most important contributions have been produced since 1970, and are by ARTHUR and MENZIES [1972], ODA [1972a, b], EL SHOBY and ANDRAWES [1973], YAMADA and ISHIHARA [1979], and again ODA [1981].

One of the most important experimental results is that virgin sand samples, generated by a deposition process, generally show a different stress-strain behaviour in the direction of deposition rather than along the bedding plane. This behaviour affects all stress-strain analyses in soils and, in particular, finite-element analysis.

This paper presents the results of an exhaustive experimental programme carried out on reconstructed sandy soil samples, with the particular aim of evaluating the geotechnical parameters characterizing the reversible elastic stress-strain behaviour of the anisotropic soil.

The experimentally determined elastic parameters were used in a finite-element model representing a rigid strip footing on elastic anisotropic soil. The results of the model were compared with those obtained from a reference elastic isotropic model, in order to show the effect of inherent anisotropy on elastic stress and settlement in the sandy soil under the footing.

2. Inherent anisotropy

Inherent anisotropy of sand, as a consequence of a sedimentation process, may be well interpreted by the geometrical model of transversal isotropy. This implies that the governing parameters are independent of horizontal directions, and the vertical axis at any point is an axis of radial symmetry (fig. 1). In this case the 21 independent elastic constants characterizing the material become five independent ones, three more than for an elastic isotropic material.

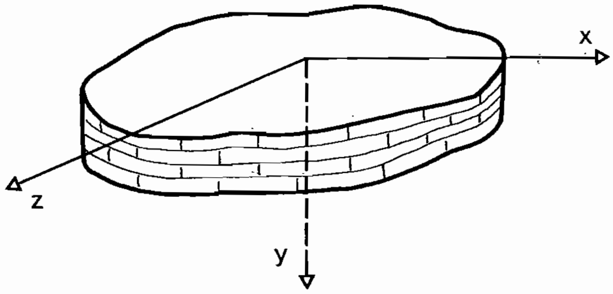

Fig. 1. - Stratified (transversely isotropic) soil.
Terreno caratterizzato da isotropia trasversale.

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Taking a system of orthogonal Carthesian coordinates XYZ and defining Y as the direction of deposition and XZ as the bedding plane, the elastic parameters may be represented by the following constants:

— E_x = Young's modulus in any horizontal direction;

— E_y = Young's modulus in a vertical direction;

— μ_{xx} = Poisson's ratio for strain in any horizontal direction due to a direct horizontal stress at right angle;

— μ_{yx} = Poisson's ratio for horizontal strain due to vertical stress;

— G_{xy} = Modulus of shear deformation in a vertical plane.

In this hypothesis, the elastic relations between stresses and strains may be explicitly espressed as follows:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} 1/E_x & -\mu_{yx}/E_y & -\mu_{xx}/E_x & 0 & 0 & 0 \\ & 1/E_y & -\mu_{yx}/E_y & 0 & 0 & 0 \\ & & 1/E_x & 0 & 0 & 0 \\ & & & 1/G_{xy} & 0 & 0 \\ & \text{symmetric} & & & 1/G_{xy} & 0 \\ & & & & & 2(1+\mu_{xx})/E_x \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} \quad (1)$$

Equations (1) contain neither μ_{xy} nor G_{xx}.
LOWE [1892] showed that:

$$\mu_{xy}/E_x = \mu_{yx}/E_y \quad (2a)$$

$$G_{xx} = E_x/[2(1+\mu_{xx})] \quad (2b)$$

Due to thermodynamics, according to which the work corresponding to any deformational process (beginning with nil deformations) shall be positive, the same matrix of elastic bonds must also be positive definite.

The values of the constants are therefore constrained as follows:

$$\begin{aligned} &— E_x > 0 \\ &— E_y > 0 \\ &— -1 < \mu_{xx} < 1 \\ &— E_x/E_y\,(1-\mu_{xx}) - 2\,\mu_{xy}^2 > 0 \\ &— G_{xy} > 0 \end{aligned} \quad (3)$$

Determination of elastic constants requires the reproduction of the following stress states:

— pure uniaxial stress state with $\sigma_x \neq 0$;

— pure uniaxial stress state with $\sigma_y \neq 0$;

— pure shear stress state with $\tau_{xy} \neq 0$.

However, sandy soil is a cohesionless material and its elastic characteristics depend on the stress state acting in it, so that reference must be made to initial stress states different from zero. Determination of elastic parameters thus requires reproduction of triaxial stress states, so Young's moduli and Poisson's ratios were experimentally determined in the laboratory using a standard triaxial cell.

Reference values, based upon elastic relations, were assigned to the shear modulus, because the great difficulty of reproducing experimentally adequate stress states with laboratory equipment.

3. Determination of elastic parameters

The use of a numerical model requires experimental determination of elastic parameter values as defined above. This may be done in the laboratory, bearing in mind two fundamental conditions:

— the soil structure must be maintained sufficiently undisturbed; it must not previously have undergone stress processes which altered its intergranular contacts;

— it is recommended to reproduce simple stress states, which may be interpreted immediately and easily reproduced by means of standard laboratory tests.

As regards the first condition, in the past many — mainly unsatisfactory — attempts at obtaining undisturbed samples of sandy soil have been made, as reported by HANZAWA and MATSUDA [1977] and MARCUSON *et al.* [1977]; because of their cohesionless nature, the samples generally showed alteration of their initial structure and intergranular contacts.

A controlled freeze-thaw cycle imparting temporary cohesion to sandy soil may be used to obtain samples for laboratory tests, since initial structural characteristics remain sufficiently unaltered. This method, described in detail in previous studies [RICCERI and SORANZO, 1981; SIMONINI, 1982], also has the advantage of allowing preparation of cylindrical soil samples with their main axis along any direction in an artificial sand deposit.

In order to describe inherent anisotropy, two particular sampling directions were chosen: the vertical or axis Y, representing the direction of the sedimentation process, and the horizontal direction, corresponding to any direction of plane XZ (bedding plane) (fig. 2).

As regards the second condition, 20 isotropically consolidated drained triaxial compression tests (C.I.D.) were carried out on medium-fine sand samples (fig. 3), reconstructed in the laboratory using the above mentioned freeze-thaw cycle. After consolidation sample density varied between 14.70 and 15.30 KN/m^3.

The sand had specific weight $G = 2.70$ and uniformity coefficient $C_u = 1.83$. Grain shapes turned out to be mainly spherical-discoidal and lamellar. Tests were carried out in a standard triaxial cell, after leaving the samples to thaw out in the cells under low confining pressure. The samples were taken to failure with a constant strain velocity of 0.07 mm/min, interposing an unloading/reloading cycle during low axial strain levels.

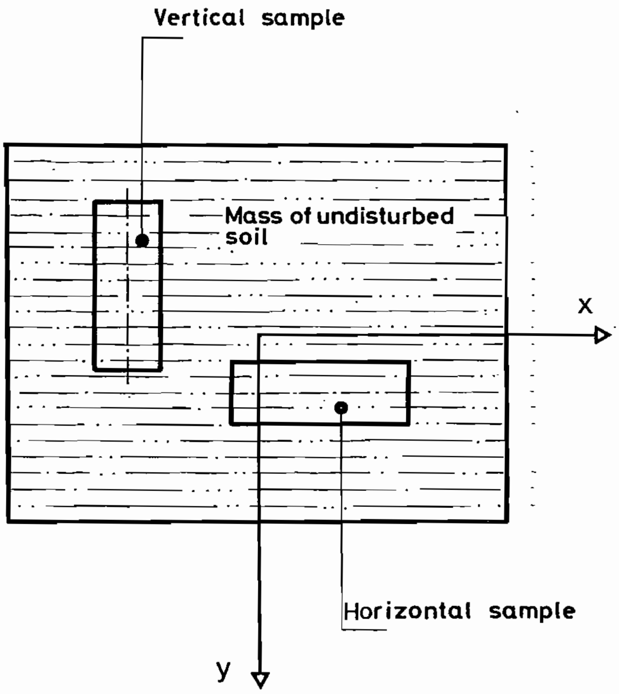

Fig. 2. - Mass of undisturbed sand with both horizontal and vertical samples.
Blocco di terreno sabbioso indisturbato con i campioni orizzontali e verticali.

Elasticity modulus

Determination of the elasticity modulus is carried out in the laboratory by means of interpretation of the relations between stresses and strains in a triaxial compression test.

Young's initial modulus is represented by the slope of the tangent at the origin of the stress-strain curve in plane (σ_1-σ_3) vs. ε_1. Its experimental determination turned out to be extremely cumbersome. It was very difficult to define the initial stretch of the curve, mainly because of initial settlement between the upper parts of the samples, porous stones, and transmission devices of axial stress. For this reason, we preferred to evaluate the elasticity modulus by means of an unloading/reloading cycle at strain levels near in situ values (fig. 4), hypothesizing that this cycle defines the elastic behaviour for each stress level [LADE, 1977]. Moreover, since sandy soil shows non-linear behaviour near monotonically increasing stress paths, the unloading/reloading modulus identifies a parameter which may be considered as fully representative of the elastic behaviour of the soil.

DUNCAN & CHANG's [1970] equation:

$$E_{ur} = K_{ur} P_a (\sigma_3/P_a)^n \qquad (4)$$

linking the unloading/reloading elastic modulus to two experimental constants K_{ur} and n and to confining pressure σ_3 was applied to the experimentally determined values of E_x and E_y, calculated from the tests carried out on horizontal and vertical samples respectively. The values of the experimen-

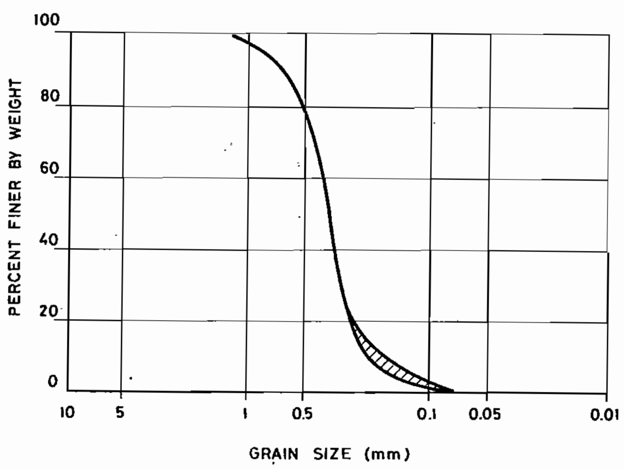

Fig. 3. - Grain-size distribution of sand.
Curva granulometrica della sabbia.

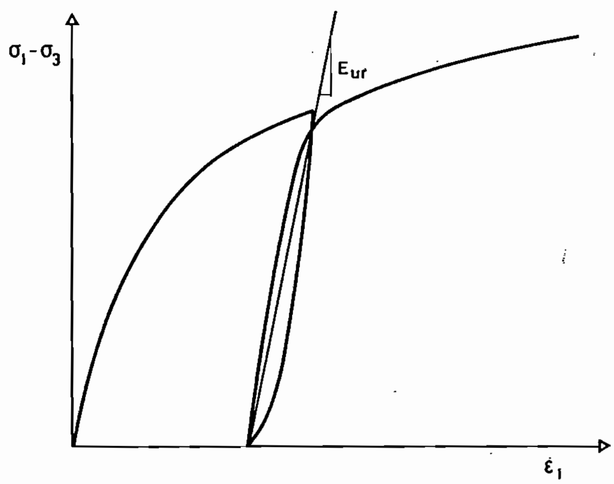

Fig. 4. - Determination of elastic modulus.
Determinazione del modulo elastico.

tal constants turned out to be $K_{ur}=1176$ and $n=0.38$ and $K_{ur}=1046$ and $n=0.40$ for the horizontal and vertical moduli respectively. Fig. 5 shows the experimental values and the interpolation lines.

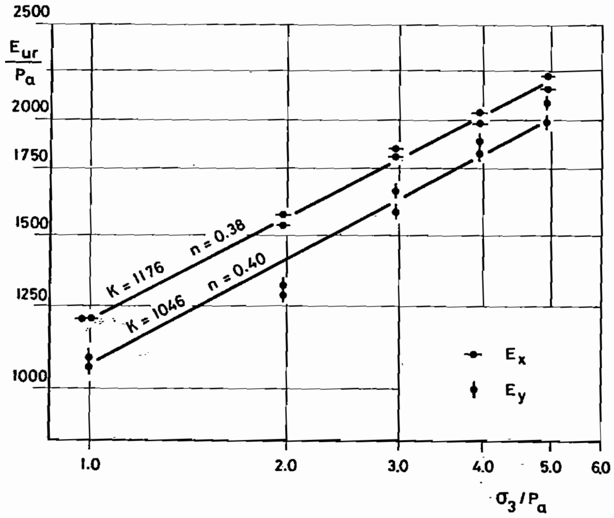

Fig. 5. - Elastic moduli as functions of confining pressure.
Moduli di elasticità in funzione della tensione di contenimento.

A comparison between the two lines shows how, at the same confining pressure as the triaxial cell, the elastic moduli of the horizontal samples are greater than those of the corresponding vertical samples.

Poisson's ratio

By definition, Poisson's ratio is the ratio between transversal and axial stresses in an axial compression test.

Poisson's ratios were again calculated corresponding to the unloading/reloading cycle, as done for the elastic moduli. The same ratio for horizontal strain produced by vertical stress was obtained by:

$$\mu_{yx} = \frac{\varepsilon_v - \varepsilon_1}{2\varepsilon_1} \qquad (5)$$

where ε_v is volumetric deformation and ε_1 axial deformation applied to the results of the tests on vertical-axis samples.

In order to calculate μ_{xx} (ratio corresponding to the horizontal strain caused by a stress belonging to the same plane), two intermediate passages were followed. First, ratio μ_{xy} linked to ratio μ_{yx} was calculated from equation 2a. The values of ratio μ_{xy} were then used in the equation:

$$\mu_{xx} = \frac{\varepsilon_v - \varepsilon_1 - \mu_{xy}\varepsilon_1}{\varepsilon_1} \qquad (6)$$

applied to the axial and volumetric deformations determined by the tests on horizontal-axis samples.

The relation:

$$\mu_{ur} = H_{ur}P_a\,(\sigma_3/P_a)^m \qquad (7)$$

was extended to both μ_{yx} and μ_{xx}. The results are shown in fig. 6. The values of the experimental constants are $H_{ur}=0.30$; $m=-0.24$, and $H_{ur}=0.18$; $m=-0.24$ for, respectively, ratios μ_{yx} and μ_{xx}.

With respect to Young's moduli, the straight lines interpolate the experimental data less well, although a reduction in with increasing confining pressure may be seen.

It should also be noted that the values of μ_{xx} are always lower than those for μ_{yx}, at the same cell pressure.

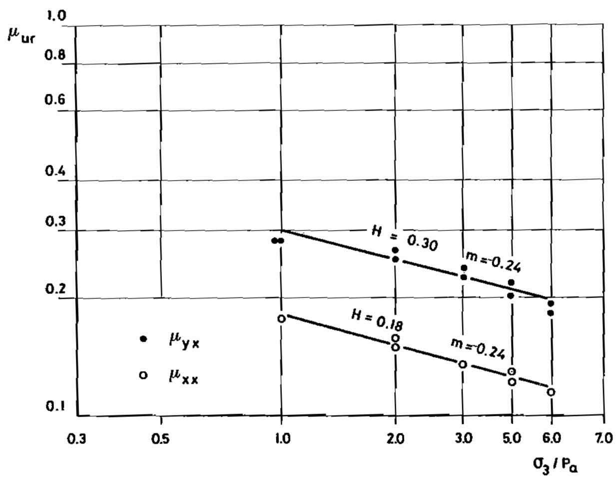

Fig. 6. - Poisson's ratios as functions of confining pressure.
Rapporti di Poisson in funzione della tensione di contenimento.

Shear modulus

Due to the difficulty in obtaining simple determinations of the shear modulus, a value determined on the basis of the vertical elastic modulus was assumed for G_{xy}. The arbitrary elastic relation used is:

$$G_{xy} = \frac{E_y}{2\,(1+\mu_{yx})} \qquad (8)$$

The shear modulus was thus correlated with the confining pressure by introducing equations (4) and (7) into equation (8).

4. Numerical model

Representation of the physical behaviour of a rigid shallow strip footing on elastic and anisotropic soil requires a model capable of translating this behaviour into analytical form. Since available closed-form elastic solutions do not seem able to define in detail the physical beaviour of the natural soil, a discrete representation was used, based on a finite-element elastic model [ZIENKEIWICZ, 1977].

Fig. 7 shows the model used for this calculation. The soil was discretized into 30 8-node isoparametric quadrilateral elements, with parabolic shape functions and numerical integration using four Gauss points..

The footing was loaded uniformly with a reference pressure of 10 KN/m^2.

Lower and lateral boundary conditions constrain the external nodes of the mesh from vertical and horizontal translation. Only horizontal translation is restricted along the central simmetry axis.

The effect of the rigid foundation is reproduced by imposing equal vertical translation on all soil/foundation interface nodes. Soil behaviour was introduced into the stiffness matrix characteristic of each element which, in the case of plane strain, takes on the form ($p = E_x/E_y$, $q = G_{xy}/E_y$):

$$\underline{D} = \frac{E_y}{(1+\mu_{xx})\,(1-\mu_{xx}-2\mu_{yx}^2)} \cdot \begin{bmatrix} p\,(1-p\mu_{yx}^2) & p\mu_{yx}(1+\mu_{xx}) & 0 \\ \text{symmetric} & (1-\mu_{xx}^2) & \\ & & q(1-\mu_{xx})\,(1-\mu_{xx}-2p\,\mu_{yx}^2) \end{bmatrix} \qquad (9)$$

Choice of the elastic constant values to be introduced into the numerical model must be made according to the stress state in the soil under the footing. Assuming that the acting vertical pressure is given by the product of the soil unit weight multiplied by depth, horizontal pressure may be linked to vertical pressure by the thrust coefficient at rest $K_o = 1 - \sin\varphi$, with equation $\sigma_x = K_o\,\sigma_y$. The stress state acting in the soil differs from that used for laboratory determination of elastic constants with C.I.D. triaxial tests. Within the limits of this application, however, variations in elastic constants could be linked to the average pressure $\sigma_y(1+2K_o)/3$ acting in the soil and the latter could be introduced into equations (7), (10) and (11). Elastic constants therefore vary with depth according to power equations, as also confirmed by GIBSON [1967] and HOLZHÖNER [1984].

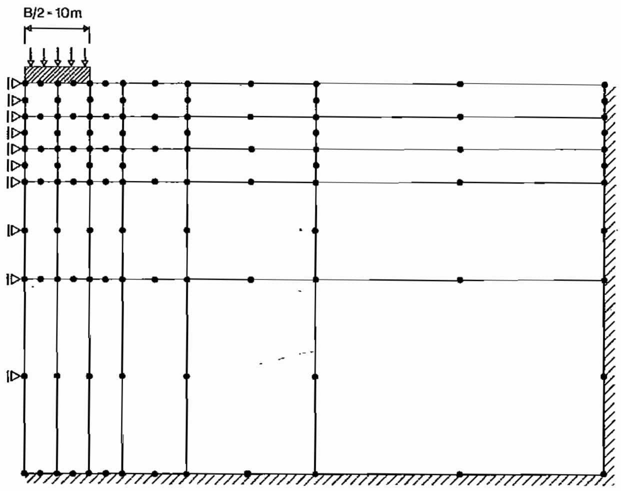

Fig. 7. - Finite element mesh.
Mesh impiegata nel modello ad elementi finiti.

The stiffness matrix of the corresponding isotropic homogeneous elastic model used for comparison

$$\underline{D} = \frac{E_y(1-\mu_{yx})}{(1-\mu_{yx})\,(1-2\mu_{yx})} \cdot \begin{bmatrix} 1 & \mu_{yx}/(1-\mu_{yx}) & 0 \\ \text{symmetric} & 1 & 0 \\ & & (1-2\mu_{yx})/[2(1-\mu_{yx})] \end{bmatrix} \qquad (10)$$

was chosen by imposing the correspondence between the vertical and horizontal elastic constants.

Numerical analysis was carried out on an IBM 4341 computer in the Faculty of Engineering of the University of Padova.

5. Results of the model

Fig. 8 shows the trend of elastic settlement at the surface and half-way down the compressible layer. No marked differences were noted in the settlement of the two types of soil examined, differences in terms of percentages being less than 2.5%. The trend of the characteristic superficial settlement for a rigid foundation changed with depth, the tendency to greater uniformity increasing from the centre outwards. Also of note was how most of the total elastic settlement was exhausted in the superficial layer of soil immediately under the foundation. In particular, the effect of inherent anisotropy clearly did not affect the calculation of reversible elastic settlement. More marked was its influence on vertical and horizontal stresses in the soil under the foundation, calculated at the centre and at the edge (figs. 9a, b and 10a, b).

Inherent anisotropy is thus shown to cause reduction in vertical and horizontal stresses on the area immediately under the foundation. In the case of horizontal stresses at the edge of the foundation, the isotropic model may show values up to 30% higher than those of the anisotropic model.

6. Conclusions

This laboratory study confirms the occurrence of inherent anisotropy in undisturbed sand samples. This type of anisotropy, easily reproduced by a geometrical model of transversal isotropy, influences the mechanical behaviour of sandy soil. In particular, the effect of inherent anisotropy may also be felt on the linear elastic component representing the reversible part of the total strain.

Elastic anisotropic constants were determined by means of a series of consolidated and drained triaxial compression tests on sand samples reconstituted in the laboratory using a freeze-thaw cycle.

Determinations were carried out by reproducing triaxial stress states on cylindrical samples taken at right angles in an artifical mass. The directions were the vertical (coinciding with that of natural sedimentation) and horizontal [coinciding with any direction in the bedding plane (or plane of transversal isotropy)].

The following general indications may be deduced from the experimental study:

— stress conditions being equal, the elasticity modulus in the isotropic plane is greater than that calculated in the normal direction;

— Poisson's ratio in the same plane is less than that calculated in any vertical plane.

The numerical values of the elastic constants, introduced into a finite-element numerical model representing a rigid shallow foundation, allowed an evaluation of the effect of inherent anisotropy measured in the laboratory on elastic settlement and on stresses in the soil under the foundation. This effect has no particular influence on the values of calculated settlement, since differences of less than 2.5% were found between the anisotropic elastic case and the corresponding isotropic case. Instead, the influence of anisotropy is found to be more marked on the vertical and horizontal stresses caused by load transmitted from the foundation. In particular, it is greater at depths, of less than half the width of the foundation.

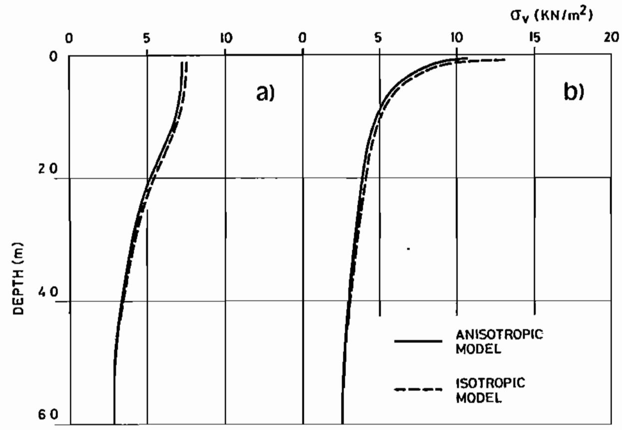

Fig. 9*a* e *b*. - Trend of vertical stresses at centre and edge of footing.
Andamento delle tensioni verticali al centro ed al bordo della fondazione.

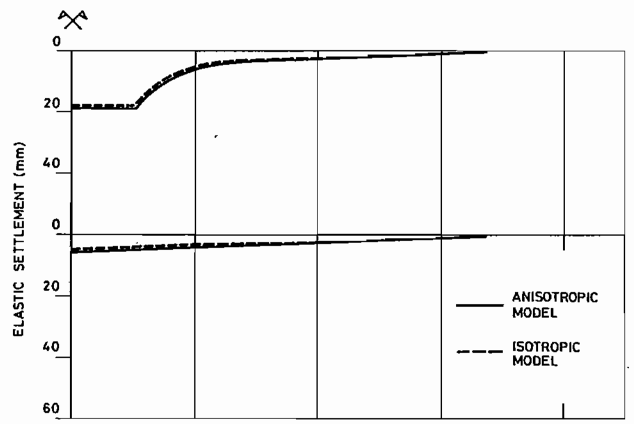

Fig. 8. - Elastic settlement at surface and half-way down compressible layer.
Cedimento elastico in superficie ed a metà dello strato compressibile.

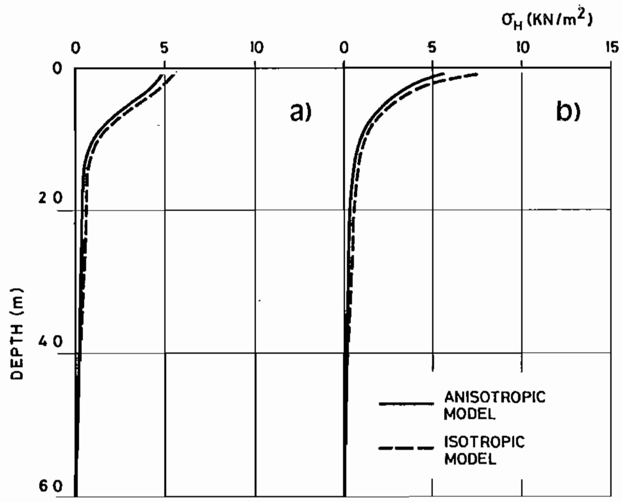

Fig. 10*a* e *b*. - Trend of horizontal stresses at centre and edge of footing.
Andamento delle tensioni orizzontali al centro ed al bordo della fondazione.

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RIASSUNTO

L'anisotropia intrinseca dei depositi sabbiosi e la sua influenza sul calcolo elastico lineare

La genesi dei depositi sabbiosi naturali viene ricondotta generalmente ad un processo di sedimentazione delle particelle di terreno granulare in un mezzo fluido. La struttura del deposito che ne risulta può essere in questo caso ricondotta ad un modello fisico di anisotropia, detta intrinseca, che viene distinta da un altro tipo di anisotropia, detta indotta, che nasce come conseguenza di processi deformativi e che quindi non è caratteristica degli ammassi indisturbati.

La modellazione dell'anisotropia intrinseca è stata oggetto di numerosi studi del passato; i contributi più significativi sull'argomento si hanno comunque a partire dal 1970. Tutti i ricercatori concordano nell'affermare che campioni vergini di terreno sabbioso, prelevati da depositi sabbiosi indisturbati, mostrano un comportamento tensionale e deformazionale diverso nella direzione parallela al piano di sedimentazione rispetto a quella normale. Ciò si riflette, in particolare, su quei parametri geotecnici che traducono in termini numerici il comportamento elastico del terreno sabbioso.

L'anisotropia intrinseca, come conseguenza di processi sedimentativi, si presta ad essere ben riprodotta da un particolare modello geometrico di anisotropia, che viene chiamato isotropia trasversale. Questo fatto implica che i parametri elastici caratteristici del terreno sono uguali nel piano orizzontale e che ogni asse verticale è un asse di simmetria radiale. In questo caso le costanti elastiche necessarie per descrivere completamente il materiale passano dalle due del caso elastico isotropo a cinque tra loro indipendenti. Esse sono:

— i moduli di elasticità E_x ed E_y;
— i rapporti di Poisson μ_{xx} e μ_{yx};
— il modulo di taglio G_{xy}

dove con y si è indicato l'asse verticale coincidente con la direzione della sedimentazione e con x un qualsiasi asse giacente nel piano xz o piano di sedimentazione.

La determinazione delle costanti elastiche richiede la riproduzione di stati tensionali caratteristici:

— uno stato monoassiale puro con $\sigma_x \neq 0$;
— uno stato monoassiale puro con $\sigma_y \neq 0$;
— uno stato tangenziale puro con $\tau_{xy} \neq 0$;

Il terreno sabbioso è però un mezzo incoerente e le sue caratteristiche di resistenza ed elasticità sono funzione dello stato di tensione agente su di esso. I moduli di Young ed i rapporti di Poisson devono quindi venire determinati imponendo stati triassiali al terreno; ciò può essere facilmente ottenuto in laboratorio impiegando un apparecchio triassiale standard.

Per la determinazione sperimentale delle costanti elastiche E_x, E_y, μ_{xx} e μ_{yx} si sono preparati, a mezzo di una sequenza controllata congelamento-scongelamento, campioni di terreno sabbioso prelevati in un ammasso, generato artificialmente, secondo due direzioni ortogonali tra loro. Queste sono la direzione verticale (y), o direzione della sedimentazione, e la orizzontale appartenente al piano di sedimentazione (xz) (fig. 2). La tecnica del congelamento dei campioni è stata impiegata perché presenta il duplice vantaggio di mantenere sufficientemente inalterata la struttura del terreno e di fornire allo stesso tempo una temporanea coesione che rende agevole la preparazione dei campioni per le prove.

Sui campioni verticali ed orizzontali, ricostituiti in laboratorio tramite la sequenza congelamento-scongelamento, si sono eseguite 20 prove di compressione triassiale consolidate isotropicamente e drenate nella cella triassiale standard. I campioni sono stati portati a rottura a velocità di deformazione costante interponendo un ciclo di scarico e ricarico in corrispondenza di contenuti valori delle deformazioni assiali.

L'esecuzione delle prove triassiali sui campioni verticali ha consentito la determinazione, in corrispondenza al ciclo di scarico e ricarico, del modulo E_y e del rapporto μ_{yx}, analogamente dai campioni orizzontali è stato possibile valutare il modulo E_x e, per via indiretta, il rapporto μ_{xx}.

Ai moduli di Young ed ai rapporti di Poisson è stata estesa la relazione di DUNCAN e CHANG (4) che lega le costanti elastiche alla tensione di contenimento σ_3 per mezzo di costanti sperimentali, che sono K_{ur} ed n per i moduli di elasticità e H_{ur} ed m per i rapporti di Poisson.

Valori di $K_{ur}=1176$, $n=0{,}38$ e $K_{ur}=1046$ $n=0{,}40$ sono stati ottenuti per i moduli elastici dei campioni orizzontali e dei campioni verticali rispettivamente. Per i rapporti di Poisson μ_{yx} sono stati trovati valori di $H_{ur}=0{,}30$ e $m=-0{,}24$ mentre per i rapporti μ_{xx} la retta interpolatrice ha fornito valori pari a $H_{ur}=0{,}18$ e $m=-0{,}24$.

A causa della difficoltà nel riprodurre con le usuali apparecchiature di laboratorio stati tangenziali puri, il modulo di taglio G_{xy}, non è stato determinato in laboratorio; ad esso sono stati assegnati valori di riferimento per mezzo della relazione elastica (8), nella quale sono state introdotte le relazioni (4) e (7) caratteristiche dei moduli E_y e dei rapporti μ_{yx}.

Dallo studio sperimentale è emerso che:

— il modulo di elasticità nel piano di isotropia è superiore a quello calcolato nella direzione normale, nelle medesime condizioni tensionali;

— il rapporto di Poisson nel medesimo piano è inferiore a quello calcolato in un qualsiasi piano verticale.

I valori dei parametri elastici sono stati impiegati in un modello numerico ad elementi finiti, rappresentativo di una fondazione superficiale rigida di tipo continuo su terreno elastico ed anisotropo. Il modello (fig. 7) utilizza elementi isoparametrici ad otto nodi con funzioni di forma paraboliche e integrazione numerica a 4 punti Gauss. Il comportamento del terreno è stato introdotto nella matrice caratteristica di ogni elemento (9) attraverso la scelta delle costanti elastiche eseguita sulla base dello stato tensionale presente nel sito prima dell'applicazione del carico. Per il legame tra lo stato tensionale presente nel terreno di fondazione ed i valori numerici delle costanti sono state assunte valide le relazioni di potenza (4), (7), e (8) nelle quali si è introdotto, al posto della tensione σ_3, il valore della pressione media agente pari a $\sigma_y (1+2K_o)/3$, dove σ_y rappresenta la tensione verticale e K_o il coefficiente di spinta a riposo.

I risultati del modello anisotropo (che si riferiscono alla particolare situazione analizzata nella quale $E_x/E_y = 1,12$ e $\mu_{xx}/\mu_{yx} = 0,60$), in termini di tensioni verticali ed orizzontali e di cedimenti elastici nel terreno sotto la fondazione, sono stati confrontati con quelli ottenuti da un modello elastico ed isotropo di riferimento (fig. 8, 9a e b, 10a e b). Dai risultati dell'analisi numerica è apparso come l'effetto dell'anisotropia intrinseca misurata in laboratorio non influisca in modo particolare sui valori del cedimento calcolato; si sono stimate differenze inferiori al 2,5% tra il caso elastico anisotropo ed il corrispondente isotropo. È risultato invece che l'influenza dell'anisotropia è più marcata sulle tensioni verticali ed orizzontali indotte dal carico trasmesso dalla fondazione soprattutto per profondità inferiori alla metà della larghezza della fondazione stessa.