Finite element kinematic limit analysis of Sallèdes trial embankment A

Khalida Sassi*, Jean-Pierre Magnan**

Summary

Kinematic limit analysis has been applied to the analysis of the trial embankment built to failure at the Sallèdes LPC experimental site. Two dimensional and three-dimensional finite element models were used. They gave promising results with respect to the applicability of this type of analysis to complicated slope stability problems. The paper briefly describes the finite element technique used, then presents 2D and 3D results.

1. Introduction

Stability analysis of natural slopes and of earth-structures built on them are generally performed by means of two-dimensional limit equilibrium methods, which assume rigid displacements of the sliding mass on top of a rigid substratum. The stability of the soil mass located above the sliding surface is judged after the ratio of resisting shear tangential forces to the shear active forces on the ground surface.

On the other hand, limit analysis offers an upper-estimate of loading forces at failure, by means of the kinematic analysis of failure mechanisms, and an under-estimate of the failure loads by means of the static analysis approach. The kinematic approach has been extensively used in France, under the influence of J. Salençon, M. Frémond, P. de Buhan, L. Dormieux, E. Leca and others. Though there is no mathematical proof that this type of analysis can give "exact" estimates of the loads that produce failure, many studies have shown that this might be the case. Besides, "engineering judgement" suggests that kinematic failure mechanisms that are similar to the movements observed in failed ground structures should give a correct value of the loads that lead to the failure. Since ground movements close to failure are usually not restricted to one plane or surface, but often include some distortion of the soil mass, it seems important, at least for research purposes, to use methods of kinematic analysis that allow for unrestricted soil movements, except at places where "real" conditions are imposed on displacements.

The finite element kinematic limit analysis approach by means of regularised Norton-Hoff model, which was first described by A. Friaâ, under the supervision of M. Frémond, later applied to soil mechanics problems by GUENNOUNI [1982] then JIANG [1992] at LCPC, is available in the programme LIMI of the general purpose finite element code CESAR-LCPC, which is under continuous development at LCPC. It has been applied to the stability analysis of natural slopes by SASSI [1996], both in two dimensional and three-dimensional conditions. Some of the results of this study are presented in this paper.

2. Principles of the F.E. regularised method of limit analysis

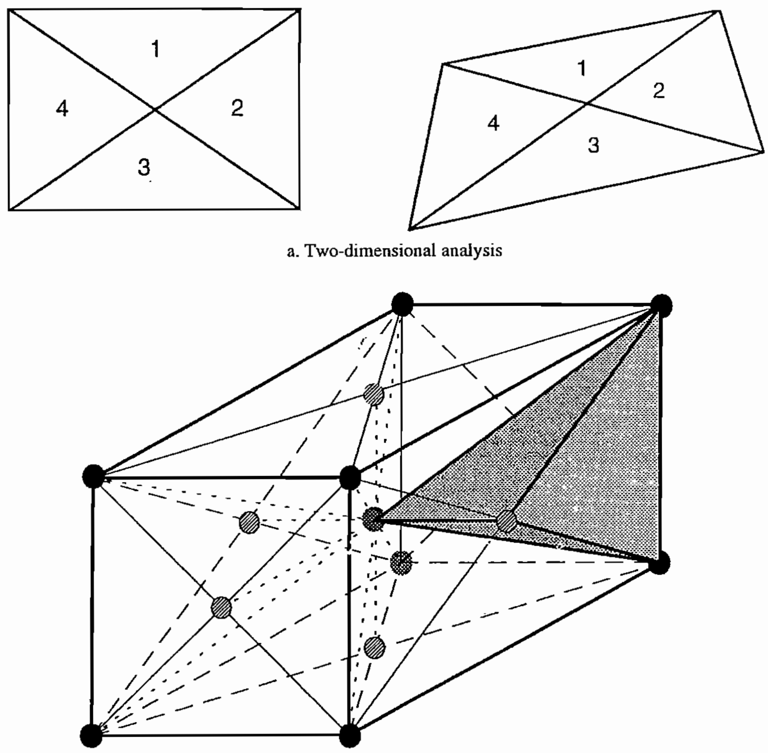
This method is based on a set of theorems proving that *"a series of visco-plastic solutions can be found that converge towards the rigid-plastic solution of the kinematic problem under study"*.

The type of visco-plastic model used is predetermined by the founding theorems (Norton-Hoff visco-plastic law). Any type of element could be used, but experience has shown that some types are more efficient (Fig. 1). A regular homogeneous finite element mesh can be used, taking merely into account the geotechnical zoning of the ground (soils layers with different properties) and the boundary conditions related to displacements, i. e. no a priori knowledge of the failure mechanism is required: the kinematics of failure will be discovered during the calculation process.

The description of the soil layers is thus restricted to their geometry and to the unit mass and failure criterion of the soil (Tresca, von Mises and Mohr-Coulomb criteria are currently used).

The procedure used for finding out the kinematic solution is described in Fig. 2: a first visco-plastic calculation is made, using a prescribed value of the viscosity parameter p of the model. The soil displacement field is determined by an iterative process, until the calculated stresses comply with the plasticity criterion. Then, the internal work rate associated with the visco-plastic strain rate is compared with the external work rate of the loads during the corresponding ground movements. The kinematic solution is said to be stable (respectively unstable) if the internal work is larger (respectively smaller) than the work of the external forces. In practice, the comparison is made by

* Ph.D. Student, Laboratoire Central des Ponts et Chaussées, Paris, France.
** Technical Director, Laboratoire Central des Ponts et Chaussées, Paris, France.



a. Two-dimensional analysis

b. Three-dimensional analysis

Fig. 1 – Efficient elements for F.E. kinematic limit analysis.
Fig. 1 – Elementi efficienti per un'analisi limite cinematica agli elementi finiti.

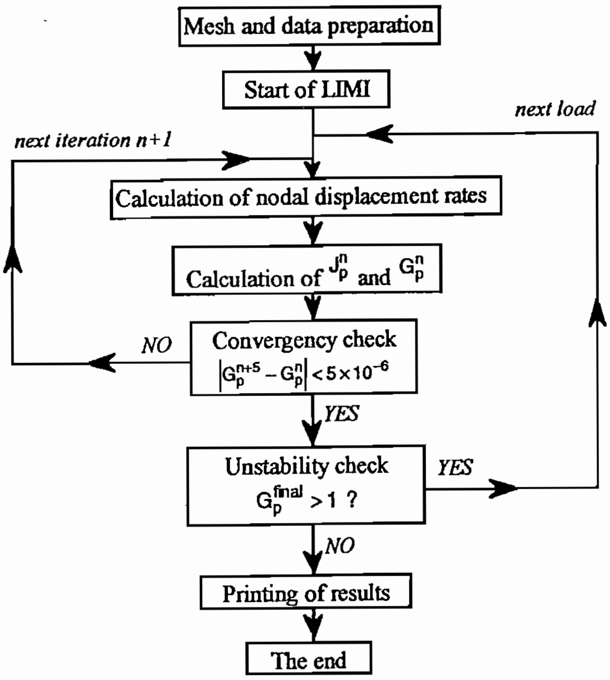


Fig. 2 – Iterative scheme for the resolution of kinematic limit analysis problems in LIMI.
Fig. 2 – Schema iterativo per la risoluzione dei problemi di analisi limite cinematica nel programma LIMI.

means of conditions put on the two functions J_p and G_p:

J_p (v) = internal work rate – external work rate

and

$$G_p = \left[\frac{pJ_p}{(p-1)\int_{\Omega} d\Omega} \right],$$

where Ω is the volume of the whole soil model.

The main criterion, for practical purposes, is the G_p value, which is less than 1 for unstable systems and higher than 1 when the ground mass is stable.

2.1. Kinematic limit analysis of Sallèdes embankment A

The French LPC experimental site of embankments on unstable slope located near Sallèdes in the Central Massif has been described by MORIN [1979], BLONDEAU *et al.* [1983] and, more recently, by ABDUL BAKI *et al.* [1993]. The geometry of embankment A is shown in Fig. 3. The stratigraphy of the slope could not be defined in a precise way, due to the variability of the soil resistance with depth, with the position of the test inside the site and with the type of test performed (Fig. 4). Therefore, various descriptions of the subsoil were used during this study.

The groundwater level in the slope happened to be a source of questions, too, due to its annual and pluriannual changes and to the limited number of

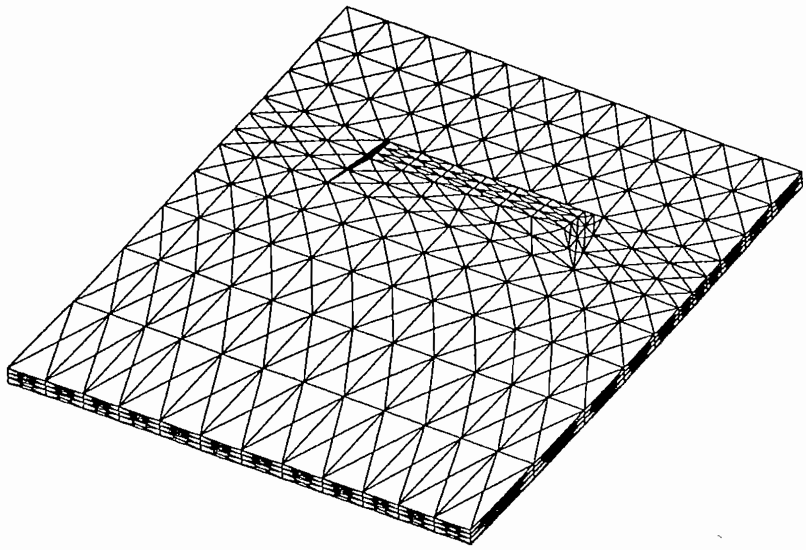


Fig. 3 – Finite elements mesh used for the three-dimensional kinemanic limit analysis.
a) Geometry: The fill is 70m long at the toe of the slope, with a 12.3 m wide platform and slopes 2/3 on the horizontal. The thickness of the fill under the front edge of the platform is 5.4 m or 5.8 m in the short term and long term calculations, respectively. The angle of the slope with respect to horizontal is 7.8 degrees.
b) Elements and nodes. The whole mesh comprises 3334 nodes and 13536 tetraedric linear elements, obtained by subdividing parallelepipedic elements with elements with 15 nodes into 24 tetraedric T4 elements.
Fig. 3 – Maglia agli elementi finiti impiegata per l'analisi limite cinematica tridimensionale.
a) Geometria: il rilevato è lungo 70 m al piede con sommità larga 12,3 m e scarpate di 2 : 3 sull'orizzontale. L'altezza è di 5.4 m o di 5.8 m nelle analisi a breve e lungo termine rispettivamente. L'inclinazione del pendio è di 7.8°.
b) Elementi e nodi: la maglia è costituita da 3334 nodi e di 13536 elementi tetraedrici lineari ottenuti suddividendo elementi parallelepipedi con 15 nodi in 24 elementi tetraedrici del tipo T4.

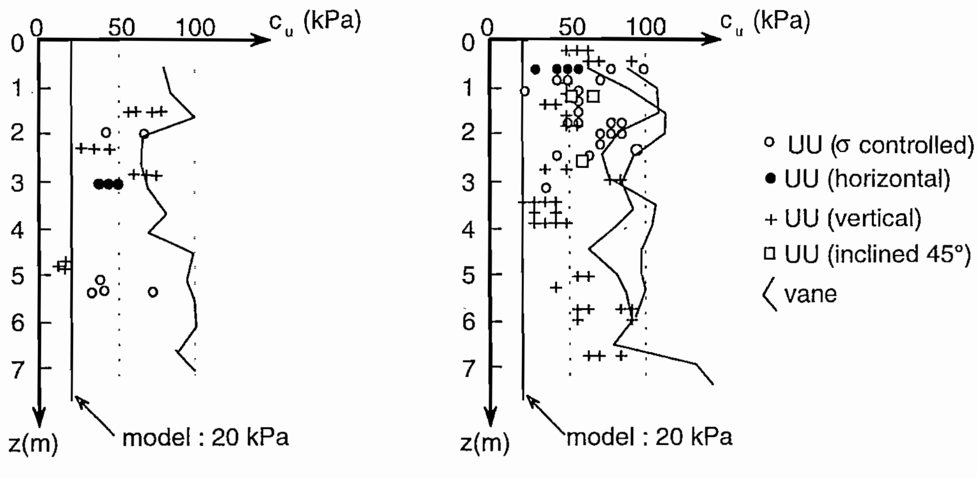


Fig. 4 – Variations with depth of the undrained shear strenght measured using various techniques in the neighbourhood of soundings C3Q4 and C0P8.
Fig. 4 – Variazione con la profondità della resistenza al taglio non drenata determinata con varie tecniche nei sondaggi circostanti il rilevato C3Q4 e C0P8.

piezometers installed in the upper, unstable layers. Some calculations were made using total stress and undrained shear strength (they are called "short term analyses" in this paper). Others were made in total stress but with drained shear resistance (zero cohesion intercept and effective angles of friction). These analyses can be interpreted as effective ones, with equivalent φ' values and are called "long term analyses" in this paper.

Some of the results of these numerical studies are presented in Figs. 5 and 6. Figure 5 shows the principal strain rates at the ground surface level (Fig. 5a) and the lines-of equal displacement rates along the three axes of co-ordinates (u towards the slope; $\dot{v}$ horizontally and $\dot{w}$ in the vertical direction, Figs. 5b,c,d, respectively) obtained in one of the short term analyses (Tresca failure criterion with undrained shear strength in all fine grained soils).

The results of a short term two-dimensional analysis are presented in figure 6. 2D-analyses enable to use more detailed meshes and thus to obtain more detailed results. As shown in Figure 6c, the failure mechanism looks like a rigid block failure along a failure surface which is very similar to the actual one [SASSI, 1996].

The best fit between the kinematic analysis and the actual behaviour at failure was obtained with an almost uniform distribution of undrained cohesion inside the slope (c_u = 20 kPa everywhere, except in the lower layer, which has a c_u = 19 kPa value; the granular fill resistance is of Mohr-Coulomb type, with c'=0 and φ' = 35 degrees).

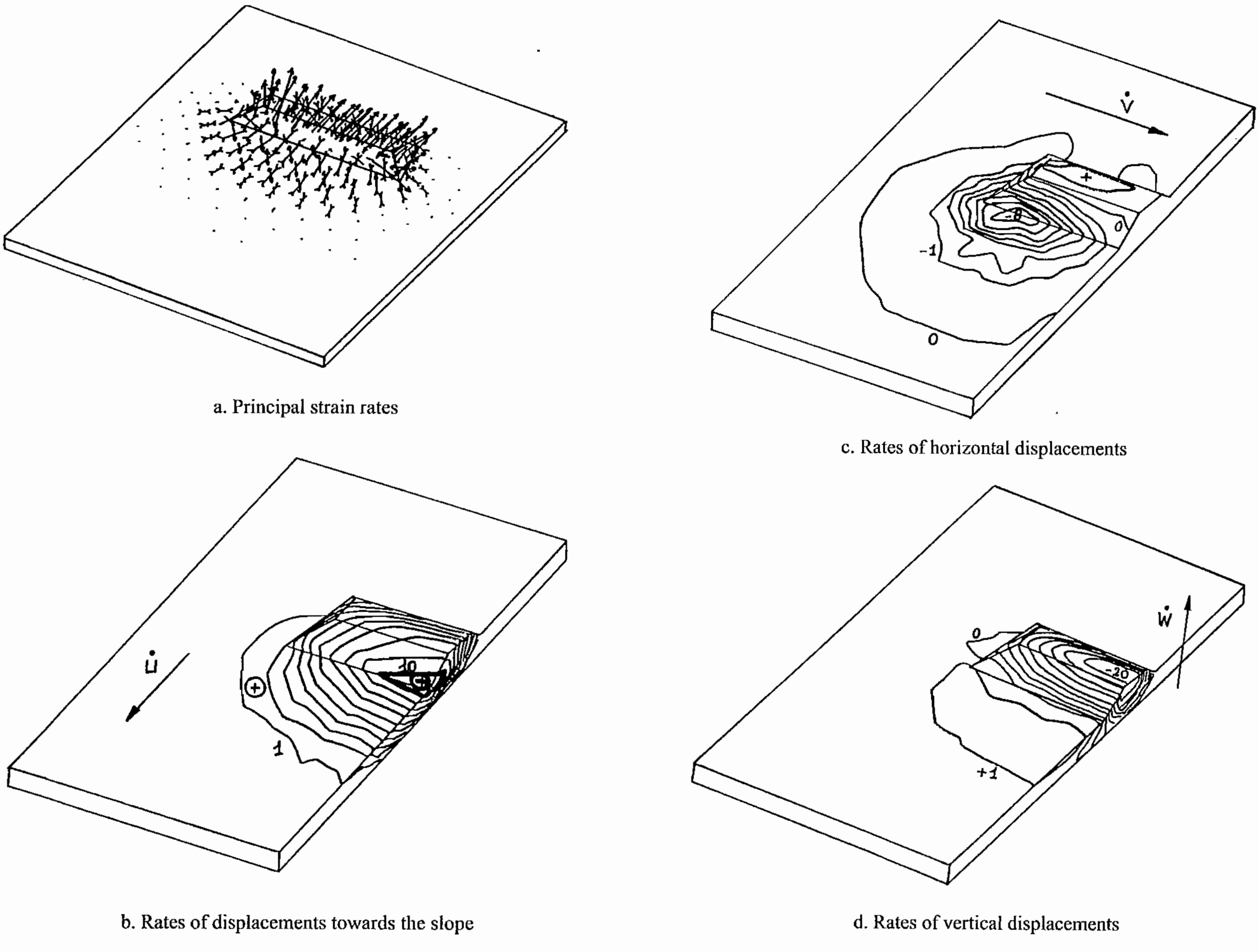
3. Conclusion

The finite element technique of kinematic limit analysis by the regularised approach has been applied to the solution of complex geotechnical stability problems for the last five years at LCPC. The results presented in this paper illustrate some of the advantages of this method: no a priori knowledge of the failure mechanism is needed and the results are quite similar to what actually happens in situ. It is therefore our intention to explore more deeply the use of this technique for slope stability analyses in cases which cannot be correctly treated by the classical two-dimensional methods of stability analysis.

References

ABDUL BAKI A., MAGNAN J.P, POUGET P. (1993) – *Analyse probabiliste de la stabilité de deux remblais sur versant instable (Site expérimental de Sallèdes, Puy-de-Dôme)*. Laboratoire Central des Ponts et Chaussées, Paris, Études et Recherches des LPC, Série géotechnique, GT 56, 199 pp.

BLONDEAU F., MORIN P., POUGET P. (1983) – *Comportement d'un remblai construit jusqu'à la rupture sur un versant naturel. Site expérimental de Sallèdes (Puy-de-Dôme)*. Laboratoire Central des Ponts et Chaussées, Paris, Rapport de Recherche LPC, n. 126, 46 pp.



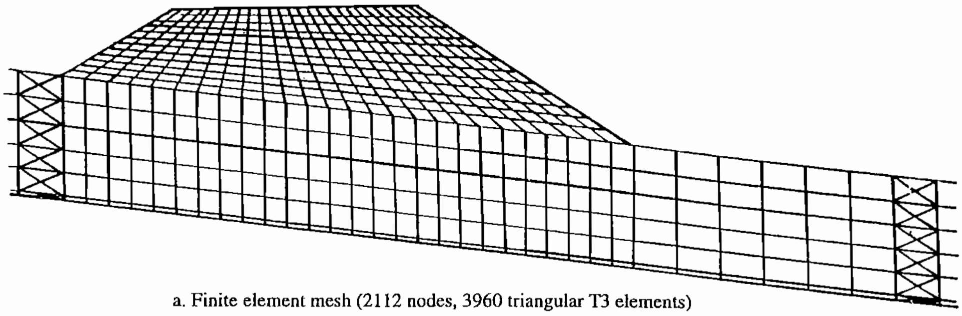
a. Principal strain rates

c. Rates of horizontal displacements

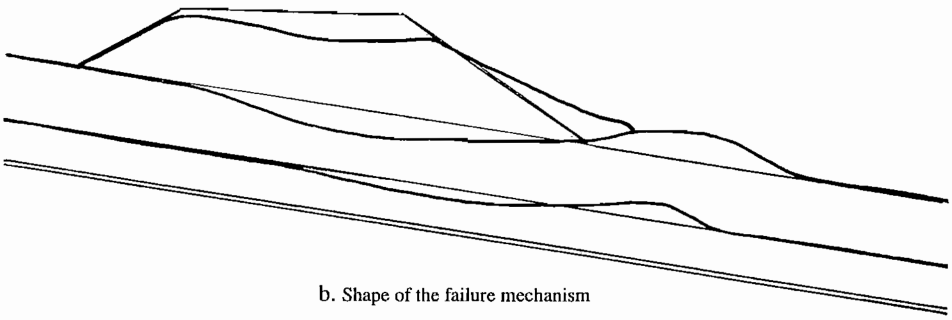
b. Rates of displacements towards the slope

d. Rates of vertical displacements

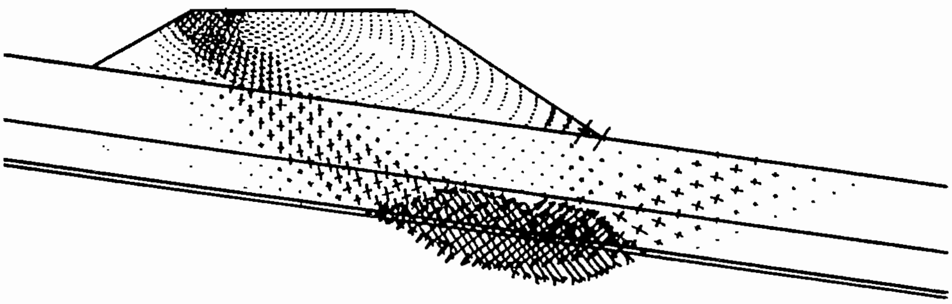
Fig. 5 – Short term kinematic limit analysis of Sallèdes embankment A.
Fig. 5 – Analisi limite cinematica a breve termine del rilevato A di Sallèdes.



a. Finite element mesh (2112 nodes, 3960 triangular T3 elements)



b. Shape of the failure mechanism



c. Principal strain rates

Fig. 6 – Two-dimensional kinematic limit short term analysis of Sallèdes embankment A.
Fig. 6 – Analisi limite cinematica a breve termine del rilevato "A" di Sallèdes: a) maglia ad elementi finiti (2112 nodi, 3960 elementi triangolari T3); b) forma della superficie di rottura; c) velocità delle defomazioni principali.

GUENNOUNI T. (1982) – *Matériau de Norton-Hoff pour divers critères de plasticité de la mécanique des sols.* Thèse de Doctorat de l'Ecole Nationale des Ponts et Chaussées, Paris, 112 pp.

JIANG G.L. (1992) – *Application de l'analyse limite à l'étude de la stabilité des massifs de sols.* Thèse de Doctorat de l'École Nationale des Ponts et Chaussées, Paris, 205 pp.

MORIN P. (1979) – *Etude du comportement avant rupture d'un remblai expérimental sur versant à Sallèdes (Puy-de-Dôme).* Thèse de Docteur-ingénieur, École Nationale des Ponts et Chaussées, Paris, 314 pp.

SASSI K. (1996) – *Contributions à l'étude des mécanismes de déformations des pentes instables.* Thèse de Doctorat de l'Institut National des Sciences Appliquées, Lyon.

Analisi limite cinematica agli elementi finiti del rilevato sperimentale "A" di Sallèdes

Sommario

L'analisi limite cinematica è stata applicata all'analisi del rilevato sperimentale dell'LPC di Sallèdes. Sono state utilizzate delle analisi agli elementi finiti bidimensionali e tridimensionali, che hanno dato dei risultati promettenti per quanto riguarda l'applicabilità di questo tipo di metodo a problemi complessi di stabilità dei pendii. Nella nota vengono descritti brevemente la tecnica agli elementi finiti usata ed i risultati delle analisi 2D e 3D.