Interaction of flexible structures with moving ground

David Muir Wood,* Jörgen Johansson**

Summary

Three examples of interaction of flexible structural elements with displacing ground are described. A closed form solution can be obtained for a pile loaded laterally at its top in an elastic soil. Where the nonlinearity of the soil-structure interaction is reckoned to be of importance appropriate functions must be assumed for the development of load on the pile. Even for this nonlinear situation the problem can be defined in terms of a small number of dimensionless groups. A third case describes a horizontal structure – a pipeline or tunnel – crossed by a fault. Understanding the material characteristics which control the analysis through dimensionless groups makes the application of the results more universal.

Keywords: soil-structure interaction; nonlinearity; elasticity; pile; numerical analysis; dimensionless group

1. Introduction

Soil-structure interaction is one of those interface topics which requires a holistic approach to the analysis or modelling. It is not possible to consider the structural and geotechnical elements of the problem separately because the system response will certainly depend on some combination of properties of both the soil and the structure. If the ground and the structure are both behaving elastically then simple configurations lead to exact analyses and results can be presented in terms of dimensionless groups which simply describe relative stiffnesses of ground and structure. The response of a laterally loaded pile is presented as an elementary example of such a closed form analysis. The resistance of the soil as the pile moves is described using a subgrade reaction model with the relative displacement of pile and ground generating resisting stress through a series of linear Winkler springs, with no allowance for the continuous nature of the enveloping ground.

In reality, however, although the structural element may behave elastically, it is extremely unlikely that the behaviour of the soil will be elastic except for extremely small deformations. So we would like to be able to extend our analysis to include non-linearity of the subgrade reaction model while still – if at all possible – retaining the possibility of displaying the results as economically as possible, making use of appropriate dimensionless groups.

* Department of Civil Engineering, University of Bristol, UK.
** Institute for Industrial Science, University of Tokyo, Japan.

The situation is often worse than this implies because we may not actually know *anything* about the nonlinear stiffness of the soil, and yet this will be crucial in deriving dimensionless groups which encapsulate sufficient detail of the parametric description of the problem. In that case we have to make informed guesses. Our expectation is that there may well be both a stiffness element to the ground-soil interaction but also a limiting interactive load. To propose a simplification of this interaction relationship is not particularly original but what is explored here is a way in which even nonlinear interaction can be capable of analytical solution and presentation in terms of nondimensional groups, thus throwing light both on the parameters which are important, and on the character of the system response.

Two related problems will be presented: the behaviour of a flexible pile in translating ground; and the effect of a fault displacement on a buried pipeline or tunnel. The geometries considered are inevitably simplistic. (A more extensive discussion of accessible soil-structure interaction analyses is given by Muir Wood [2004] which expands the presentation of the first of these two systems.)

2. Pile under lateral loading

The first problem is the familiar elastic soil-structure interaction problem of a vertical pile under lateral loading at the ground surface (Fig. 1). The geometry and loading are deliberately kept simple. The soil is elastic, the pile is of width *B*, has flexural rigidity *EI*, and is loaded by a lateral force *P* at ground surface. The lateral pressure generated between the soil and the pile is assumed to be

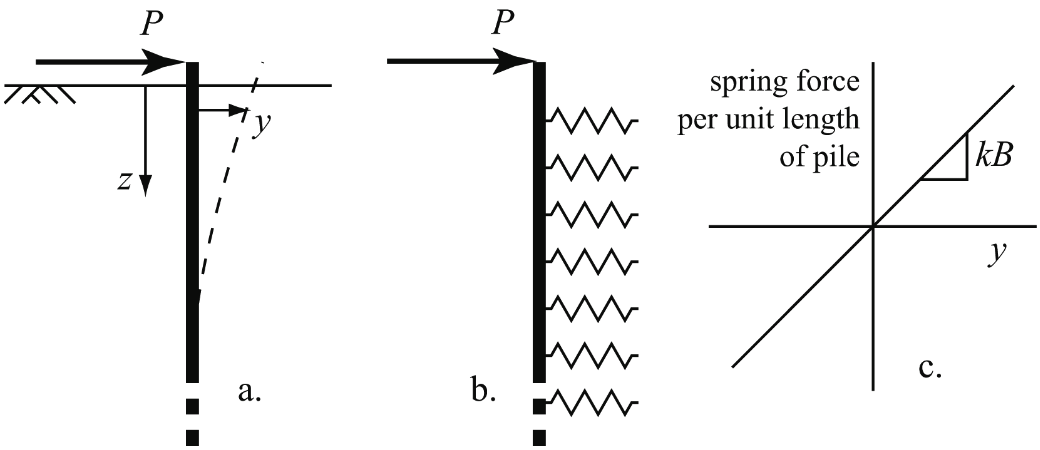

Fig. 1 – (a) Laterally loaded pile in elastic soil; (b) representation of soil by Winkler springs; (c) linear response of Winkler springs.

Fig. 1 – Palo sollecitato da una forza orizzontale in testa in terreno elastico; (b) schematizzazione del terreno mediante il modello di Winkler; (c) risposta lineare delle "molle" di Winkler.

directly proportional to the relative movement of the pile and the soil according to a coefficient of horizontal subgrade reaction k. The governing equation of flexural equilibrium of the pile is:

$$EI\,\frac{d^4y}{dz^4} = -kBy \qquad (1)$$

where the lateral displacement is y at coordinate z, measured from the ground surface. The general solution is

$$y = e^{\mu z}(A_1\cos\mu z + A_2\sin\mu z) + e^{-\mu z}(A_3\cos\mu z + A_4\sin\mu z) \qquad (2)$$

or alternatively

$$y = (B_1\cosh\mu z + B_2\sinh\mu z)(B_3\cos\mu z + B_4\sin\mu z) \qquad (3)$$

where

$$\mu^4 = \frac{kB}{4EI} \qquad (4)$$

and where A_1, A_2, A_3, A_4 or B_1, B_2, B_3, B_4 must be determined from the boundary conditions of a particular problem.

If the pile is infinitely long, then the boundary conditions are: at $z = 0$, moment $M = EId^2y/dz^2 = 0$ and shear force $F = EId^3y/dz^3 = P$; and as z ♦ ×, moment $M = EId^2y/dz^2$ 0 and shear force $F = EId^3y/dz^3$ ♦ 0. The solution is

$$y = \frac{2P\mu}{kB}\,e^{-\mu z}\cos\mu z \qquad (5)$$

The variation of deflection, slope, moment and shear force down the pile is shown in Figure 2. The pile develops an undulating deflected form as the load is transferred down the pile: not much of significance happens below about $z \cup 6/\mu$. So we might propose that the pile would behave similarly to a pile of finite length l, built in at its end, provided $\mu l > 6$ or $kBl^4/4EI > 6^4 = 1296$.

For a pile of finite length the boundary conditions are: at $z = 0$, moment $M = EId^2y/dz^2 = 0$ and shear force $F = EId^3y/dz^3 = P$; and at $z = l$, moment $EId^2y/dz^2 = 0$ and shear force $EId^3y/dz^3 = 0$. The solution is

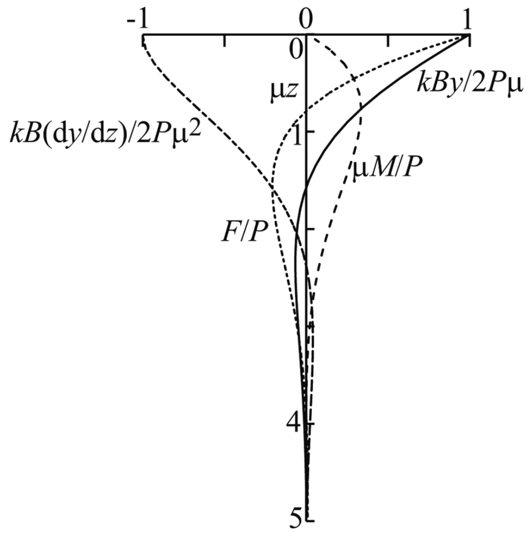

Fig. 2 – Infinite laterally loaded pile in elastic soil: normalised displacement y, slope dy/dx, moment M and shear force F.

Fig. 2 – Palo di lunghezza infinita sollecitato da una forza orizzontale in testa in terreno elastico: andamenti dello spostamento normalizzato $kBy/2P\mu$, dell'inclinazione normalizzata $kB(dy/dz)/2P\mu^2$, del momento normalizzato $\mu M/P$ e del taglio normalizzato F/P.

$$\frac{kBly}{2P} = \frac{\mu l}{\sin^2 \mu l - \sinh^2 \mu l} \times$$

$$\left[\sinh^2 \mu l \sinh \mu z \cos \mu z + \sin^2 \mu l \cosh \mu z \sin \mu z + \right. \quad (6)$$

$$\left. (\sin \mu l \cos \mu l - \sinh \mu l \cosh \mu l) \cosh \mu z \cos \mu z \right]$$

The dimensionless parameter that controls the behaviour of the pile is μl or $\mu^4 l^4 = kBl^4/4EI$ which is a function of *relative* stiffness of pile and soil. Typical normalised displacements and moments for values of μl between 1 and 4 are shown in Figure 3. For a stiff pile (low values of μl) the pile hardly bends at all but kicks backwards in order to generate the moment to resist the applied load. For more flexible piles the lateral deflection of the top of the pile increases and the flexure of the pile also increases.

3. Pile in displacing ground

Situations in which both the structural and the soil materials can be described as linear elastic lend themselves to closed-form analysis. However, the behaviour of soils is at best linear elastic to very small shear strains and the subsequent nonlinearity will certainly have an influence on the nature of the soil-structure interaction. In this section we will investigate some aspects of the structural consequences of slow movement of ground past a pile. We will model the interaction of the pile with the soil by means of a series of *nonlinear* springs – and explore the effects of this more realistic description.

The stimulus for performing this analysis was provided by a prototype problem in which a landfill through which a piled structure had been constructed was known to be sliding slowly but inexorably down the shallow slope of its underlying rock surface. There are two major unknowns: the profile of lateral displacement within the fill is not known; and the nature of the (nonlinear) fill-structure interaction is not known. It is nevertheless possible to introduce some fairly rational descriptions of these two aspects of the interaction and then perform parametric studies. To put it further in context, the structural interest was to understand the extent to which knowledge of the lateral displacement of the piles at the 'ground' surface could be used to characterise the maximum moment generated in the piles.

The analysis of active or passive lateral loading of piles using a set of subgrade reaction springs is not novel [REESE and MATLOCK, 1956; POULOS, 1973] but such analyses have usually assumed that the soil is elastic, or piecewise linear (though Poulos introduces the possibility of a limiting soil resistance). The problem to be analysed here is approached in a different way.

The piles are of length l, and flexural rigidity EI, and are fixed at their base, so that both rotation and displacement are prevented (Fig. 4). A dimensionless coordinate $\eta = z/l$ defines position on the pile with $\eta = 0$ at the base and $\eta = 1$ at the ground surface. The pile has no restraint at the top, $\eta = 1$ and, for the purposes of this analysis, it is assumed that the only lateral loading is provided by the moving fill.

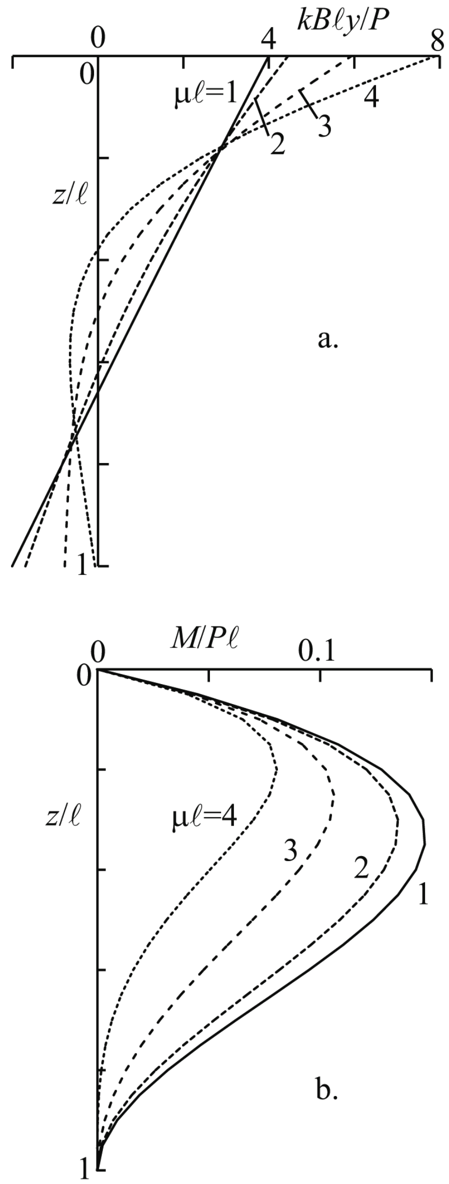

Fig. 3 – Laterally loaded pile of length l in elastic soil: influence of relative pile stiffness μl: (a) normalised displacements and (b) normalised moments.
Fig. 3 – Palo di lunghezza sollecitato da una forza orizzontale in testa in terreno elastico: influenza della rigidezza relativa μ del palo: a) spostamenti normalizzati, e b) momenti normalizzati.

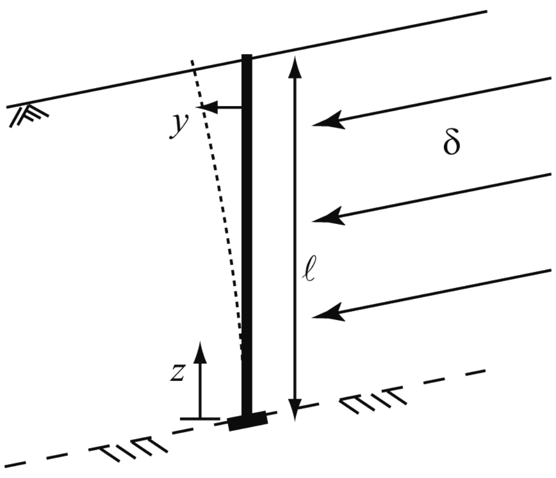

Fig. 4 – Pile loaded by translating ground.
Fig. 4 – Palo sollecitato da terreno in movimento traslatorio.

We do not know the detail of the profile of ground movement so we give it a general profile:

$$\delta = \delta_o \eta^{\alpha} \tag{7}$$

where δ_o is the movement at the ground surface, at $\eta = 1$, and α is a parameter which characterises the profile of movement (Fig. 5). A value of $\alpha = 1$ implies linear variation of movement with depth; $\alpha > 1$ implies that the movement is more concentrated towards the surface (in principle $\alpha = \times$ implies that movement is everywhere zero except for $\eta = 1$); $\alpha < 1$ implies that the movement is more concentrated towards the base of the ground ($\alpha = 0$ implies that the ground is moving as a block with $\delta = \delta_o$ at all depths). It is assumed that the presence of the individual piles does not influence the 'free-field' flow of the ground.

The detail of load generation through relative movement of ground and structure is not known – and the fact that the 'ground' in this case is actually landfill does not help. We can make some reasonable propositions. The ground-structure interaction is likely to be symmetric and dependent only on the *magnitude* of the relative movement. As relative movement increases it is expected that the load generated will saturate to some limiting value characterised by a strength property of the ground. It is assumed that this strength varies linearly with depth. There will be some description of the stiffness of the interaction at small relative displacements but the tangential, incremental stiffness will fall as the limiting strength is approached. An appropriate expression for the lateral pressure on the pile, as a function of relative displacement $\Delta = \delta - \gamma$ could then be (Fig. 6):

$$\frac{K}{K^*} = \tanh\left[\lambda\left(\frac{\Delta}{B}\right)\right] = \tanh\left[\beta(\eta^{\alpha} - \zeta)\right] \tag{8}$$

where $\zeta = \gamma/\delta_o$ and $\beta = \lambda\delta_o/B$. The first part of this equation indicates that the pressure on the pile depends on the relative movement Δ of pile and soil (which can be positive or negative) normalised by a typical pile dimension B related to pile diameter or pile width. The 'stiffness' of the pile:ground relationship is controlled by λ. The mobilised lateral pressure coefficient K is linked through the nonlinear stiffness with a limiting asymptotic lateral pressure coefficient K^* reached as the relative movement increases. Thus λ and K^* are subgrade reaction parameters.

The second part of the equation normalises the relative displacement of pile and ground with the displacement δ_o of the ground at the surface, as in (7). That then leads to the dimensionless group

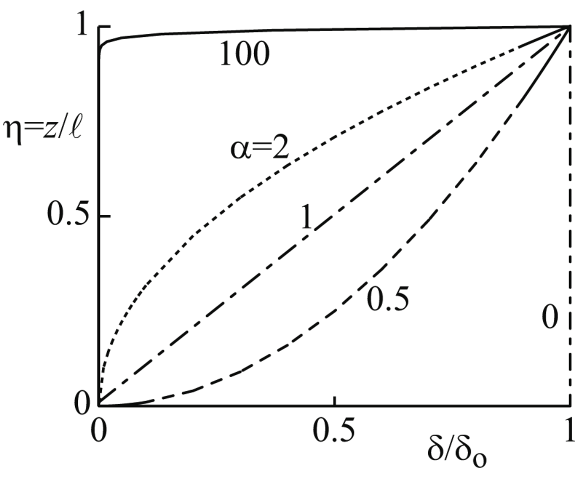

Fig. 5 – Profiles of ground displacement.
Fig. 5 – Profili di spostamento del terreno.

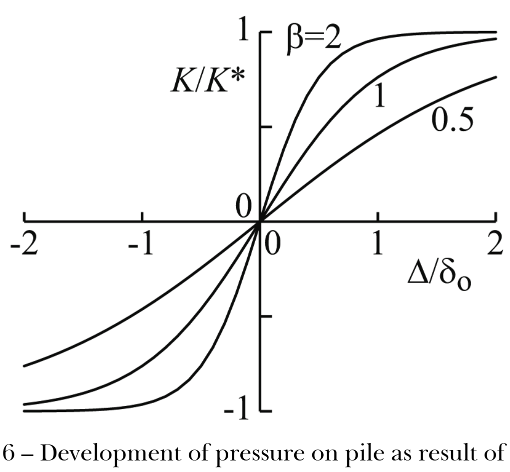

Fig. 6 – Development of pressure on pile as result of relative movement between pile and ground.
Fig. 6 – Sviluppo delle spinte sul palo a seguito dei movimenti relativi tra palo e terreno.

$\beta = \lambda\delta_o/B$ which combines information about the stiffness of the pile-ground response, λ, with a ratio of length parameters: ground movement at the free surface, δ_o and typical pile dimension, B.

The earth pressure coefficient K is applied to the vertical stress at any depth on the pile in order to calculate the horizontal stress on the pile generated by the relative movement between pile and soil.

$$\sigma_h = K\gamma l\,(1-\eta) \tag{9}$$

where γ is an appropriate unit weight for the soil.

For a material with frictional strength the limiting earth pressure coefficient K^* would be linked with the passive pressure coefficient K_p for the soil. In a cohesive material the undrained strength, and hence the limiting pressure, might be assumed to be constant over the length of the pile: this is the assumption made in the next section for a different structural configuration. The tanh function is symmetric and will behave identically whatever the sign of the relative movement between ground and pile. The initial stiffness of the pile-ground interaction can be presented as a coefficient of subgrade reaction (stress per unit relative displacement of pile and ground):

$$k_h = \frac{d\sigma_h}{d\Delta} = \lambda K^*\gamma\,\frac{l}{B}(1-\eta) \tag{10}$$

with the value $k_{ho} = \lambda K^*\gamma l/B$ at the base of the pile ($\eta = 0$). The relative movement required to move halfway to the limiting value (corresponding to $K/K^* = 1/2$) is $\Delta_{50} = (B\ln 3)/(2\lambda) = (\delta_o \ln 3)/(2\beta)$ or

$$\beta = \frac{\ln 3}{2}\,\frac{\delta_o}{\Delta_{50}} \tag{11}$$

which provides a potential link between β and other parameters describing the problem. We assume that the lateral stress σ_h acts over a width B related to the diameter of the pile. Deformation of the pile is then governed by the dimensionless equation

$$\frac{d^4\zeta}{d\eta^4} = \frac{1-\eta}{\chi}\,\frac{K}{K^*} = \frac{1-\eta}{\chi}\tanh\left[\beta(\eta^\alpha - \zeta)\right] \tag{12}$$

with

$$\chi = \frac{EI\delta_o}{l^2}\,\frac{1}{K^*\gamma l^3 B} \tag{13}$$

The problem is thus controlled by three parameters: α describes the profile of ground movement; β controls the initial stiffness of interaction; and χ is the ratio of two moments, one a structural property and the other a loading characteristic. For a cantilever of length l and flexural rigidity EI subjected to a tip load which produces a tip displacement δ_o, and subjected to no other loading, the root moment is $M_r = 3EI\delta_o/l^2$ For a cantilever of length l subjected to lateral pressures over a width d given by the limiting value of the lateral stress coefficient $K = K^*$ over the entire length, the root moment is $M_f = K^*\gamma l^3 B/6$. Thus $\chi = M_r/18M_f$.

The boundary conditions for the cantilever pile shown in Figure 4 are zero deflection and slope at the base of the pile (assuming complete fixity at the base) : $\zeta = d\zeta/d\eta = 0$ for $\eta = 0$; and zero moment and shear force at the top of the pile $d^2\zeta/d\eta^2 = d^3\zeta/d\eta^3 = 0$ for $\eta = 1$.

Once the deformation Equation (12) has been solved to give a profile of normalised displacement ζ with normalised depth η, the variation of moment M within the pile can be presented in various dimensionless ways:

$$\mu = \frac{d^2\zeta}{d\eta^2} = \frac{M}{EI\delta_o/l^2} \tag{14}$$

normalises the moment with input parameters of the problem. The displacement of the tip of the pile is an output quantity, $y_{max} = \zeta_1\delta_o$, where $\zeta = \zeta_1$ at $\eta = 1$, which will in general be different from the ground displacement δ_o. A cantilever in air whose tip is moved sideways by a distance y_{max} develops a root moment $M_{max} = 3EIy_{max}/l^2$ so that the dimensionless group μ_1

$$\mu_1 = \frac{M}{M_{max}} = \frac{\mu\delta_o}{3y_{max}} = \frac{\mu}{3\zeta_1} \tag{15}$$

allows us to compare the moments in a pile whose lateral movement is brought about by the translation of the ground with the maximum (root) moment in a pile in air given the same movement at the top, and thus allows us to understand the extent to which interpretation of maximum moment from the observed displacement of the piles at the ground surface is or is not unconservative. On the other hand the dimensionless group μ_2:

$$\mu_2 = \frac{M}{M_f} = \frac{M}{K^*\gamma l^3 B/6} = 6\chi\mu \tag{16}$$

normalises the moment with the maximum moment that can be generated when the soil is slipping past the pile and fully mobilising the resistance coefficient K^* over the full length of the pile and thus indicates the progress of moment mobilisation.

The two parameters, β and χ, both involve the surface movement, δ_o, of the ground: parameter β describes the stiffness of the pile-ground interaction and χ introduces the pile flexibility. For a given pile the ratio:

$$\frac{\beta}{\chi}=\frac{\ln 3}{2}\frac{\delta_o}{\Delta_{50}}\frac{l^2}{EI\delta_o}\left(K^*\gamma l^3 B\right)=\frac{\ln 3}{2}\frac{K^*\gamma l^5}{EI}\frac{B}{\Delta_{50}} \qquad (17)$$

forms a composite parameter which introduces pile flexibility, limiting earth pressure ratio, and the stiffness of pile-ground interaction. We can investigate the behaviour of the pile-ground system by varying β and χ both separately and together. It is therefore helpful to have some baseline values for the parametric study.

The definition of β introduces both the relative movement Δ_{50} required to generate half the limiting load (a key element of the ground-pile interaction), and the magnitude of the ground surface movement δ_o (which is completely independent of the pile response). Perhaps Δ_{50} might be about half the pile diameter – we could in principle perform tests to discover its value. The ground surface movement is quite unknown but as an order of magnitude one might suppose it to be of the same order as the pile diameter. Then, for pile diameter $B=0.5$ m ($=2\Delta_{50}$), $\beta=1.1$. A reference value $B=1.0$ thus seems reasonable. Stiffer pile-ground interaction (lower Δ_{50}, more rapid attainment of the limiting pressure) will imply increased β. An earlier stage in the development of ground movement after installation of the piles will imply lower δ_o and hence a decreased value of β.

A reference value of χ can be estimated in the same way. We have already assumed an order of magnitude of ground movement $\delta_o=0.5$ m. We will suppose that the piles are of diameter: $B=0.5$ m (implying $I=\pi B^4/64$∪0.0031 m^4); and that the Young's modulus for concrete: $E=30$ GPa. We also need to choose a pile length: $l=10$ m. The unit weight of the ground, which drives the development of lateral stress on the pile is: $\gamma=10$ kN/m^3; and we might take the limiting lateral pressure coefficient: $K^*=2$ (corresponding to the Rankine passive pressure ratio K_p for angle of friction ϕ'∪20°). Then $\chi=0.0464$. A reference value $\chi=0.05$ has been used for parametric studies and the effect of increasing and decreasing this has been explored over the range 0.0004 to 0.1. The value of χ is very sensitive to the details of the pile geometry: $\chi\sim B^3(\sim I/B)$ and l^{-5} (see (13)).

The governing Equation (12) is extremely nonlinear but is capable of numerical solution using standard routines. A short program has been written in MATLAB to apply the solver bvp4c to the fourth order governing differential equation. Results are shown in plots of dimensionless pile displacement $\zeta=y/\delta_o$ (Figs. 7 and 8) and of dimensionless moment μ_1 (Figs. 9, 10) with dimensionless position on the pile $\eta=z/l$.

The effect of varying α is as expected (Figs. 7a): the greater the average movement of the ground (the lower the value of α) the greater the load on the pile and the greater the pile movement and root moment. The value of α is not something over which an engineer has much control, but any tendency for mass movement of the ground to occur will certainly be very damaging for any structure getting in the way of the motion. A baseline value of $\alpha=1$, corresponding to linear variation of ground translation with depth, has been used for most analyses.

The pile displacements and moments are very sensitive to the stiffness of the interaction between the pile and the ground: the higher the value of β the larger the displacements and moments

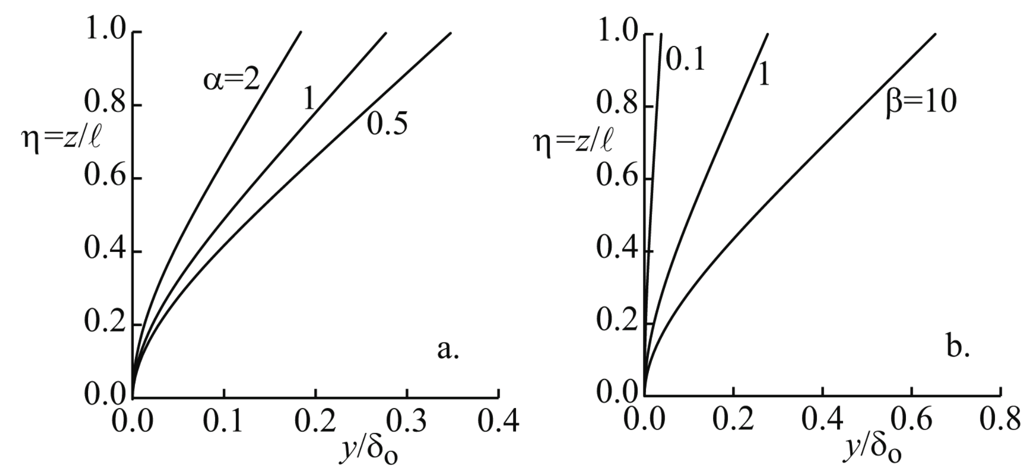

Fig. 7 – Deflected shape of pile for (a) different values of α ($\beta=1$, $\chi=0.05$); (b) different values of β ($\alpha=1$, $\chi=0.05$).
Fig. 7 – Configurazioni deformate del palo per (a) diversi valori di α ($\beta=1$, $\chi=0.05$); (b) diversi valori di di β ($\alpha=1$, $\chi=0.05$).

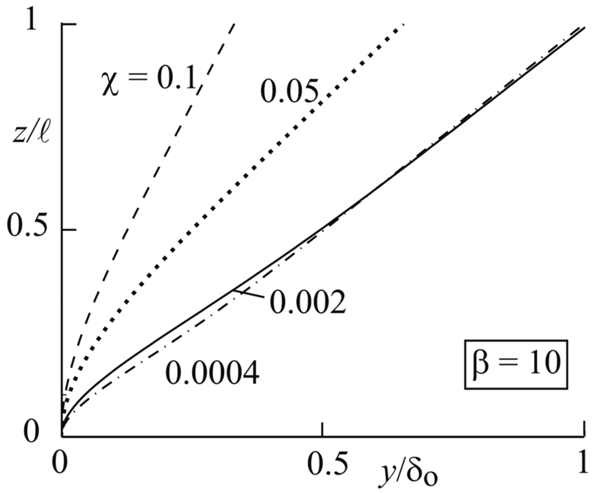

Fig. 8 – Deflected shape of pile for different values of χ (α = 1, β = 10).
Fig. 8 – Configurazioni deformate del palo per diversi valori di χ (α = 1, β = 10).

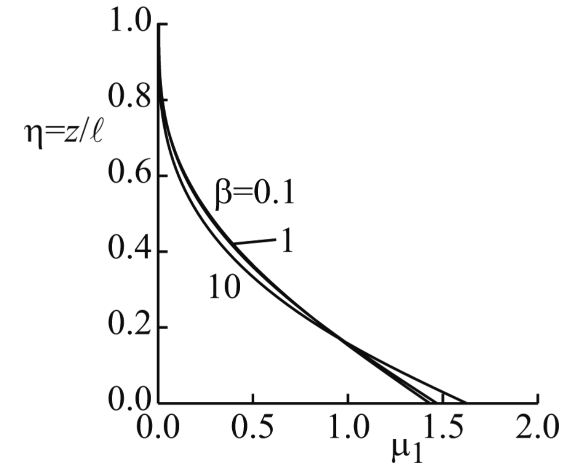

Fig. 9 – Normalised moments in pile for different values of β(α = 1, χ = 0.05).
Fig. 9 – Momenti normalizzati lungo l'asse del palo per diversi valori di β (α = 1, χ = 0.05).

(Figs. 7b and 9). However, there is interaction between the effects of changing β and χ, which is dependent on the pile stiffness. With a lower value of $\chi = 0.002$, the tip deflection of the pile is hardly affected by the value of β for $\beta > 1.0$ although the greater curvature at the toe of the pile leads to much higher moments (Fig. 8).

Increase of pile stiffness through χ has the expected effect of reducing pile deflection (Fig. 8). However, reducing χ below 0.002 has little additional effect on the pile displacements: the pile is sufficiently flexible that the displacement at the ground surface matches the displacement of the ground itself. The profile of dimensionless moment, μ_1, remains unchanged for values of pile stiffness $\chi > 0.05$.

The interaction of values of β and χ in influencing the tip movement and root moment is shown in Figure 11. It is clear that for a very flexible pile ($\chi = 0.0004$) the tip movement is more or less equal to the ground movement for all values of β (Fig. 8), whereas for less flexible piles the proportion of ground movement at the tip increases with β (or, which is equivalent, with δ_o). Similarly, the scaled moment, μ_1, is independent of β for higher values of pile stiffness (the stiffer pile forces the ground to reach its limiting pressure as it flows past the pile). For the most flexible pile considered the maximum moment is nearly 6 times the free air value; for the stiffer piles it is still more than 50% higher than the value obtained for a cantilever displaced in free air. Thus one of the questions posed is clearly answered: soil-structure interaction leads to higher moments at depth (out of sight) than would be estimated from observing the ground surface displacement of the pile and treating it as an object in air. The result is of course obvious and intuitive but requires acknowledgement of the interaction between soil and structural properties. It can be concluded that observation of pile tip movement at the ground surface gives a very poor indication of the magnitude of moments in the pile.

Since the two dimensionless parameters β and χ both contain the surface ground movement δ_o, the process of gradual mobilisation of ground-pile interaction for a given pile can be followed by varying *both* β and χ in appropriate constant proportion.

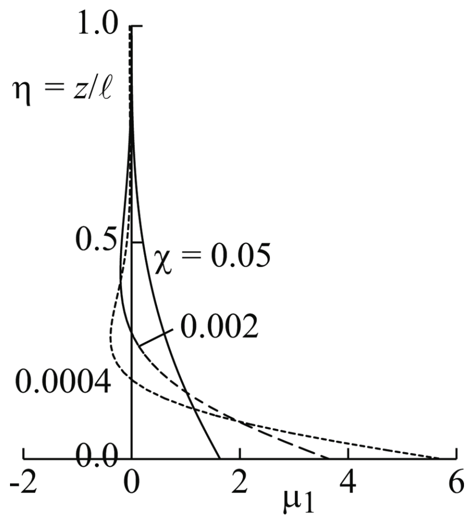

Fig. 10 – Normalised moments in pile for different values of χ (α = 1, β = 10).
Fig. 10 – Momenti normalizzati lungo l'asse del palo per diversi valori di χ (α = 1, β = 10).

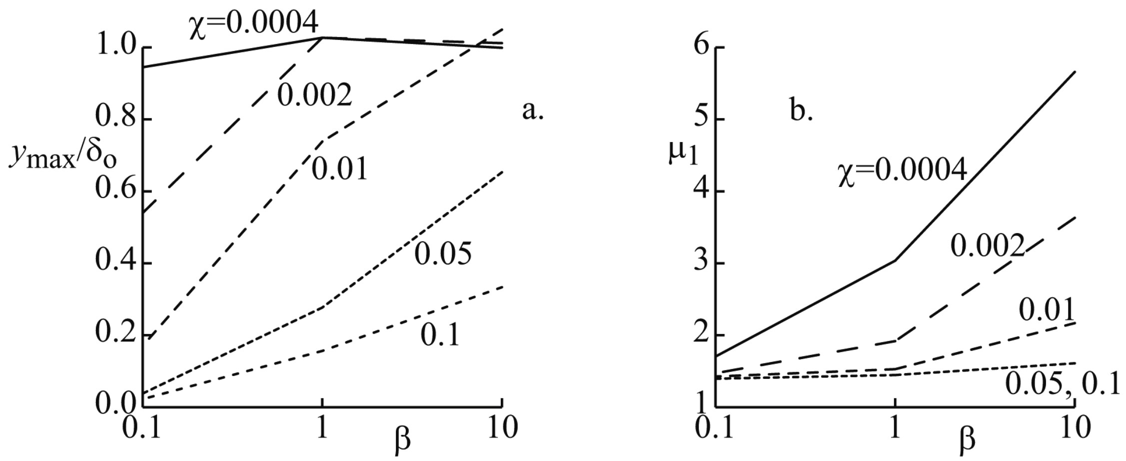

Fig. 11 – Influence of β and χ on (a) maximum deflection and (b) maximum moment developed in pile.
Fig. 11 – Influenza di β e χ su (a) massimo spostamento normalizzato e (b) massimo momento normalizzato.

$$\frac{\beta}{\chi}=\frac{\ln 3}{2}\frac{\delta_o}{\Delta_{50}}\frac{l^2}{EI\delta_o}\left(K^*\gamma l^3 B\right)=\frac{\ln 3}{2}\frac{K^*\gamma l^5}{EI}\frac{B}{\Delta_{50}} \quad (18)$$

and for the typical values that we have suggested here, $\beta/\chi \cup 20$. We can follow the gradual mobilisation of pile moment by looking at the variation of the normalised moment μ_2 (16) with increasing normalised ground displacement $\delta_o/B \cup \beta$ (Fig. 12). As ground displacement builds up this normalised moment approaches 1: the ground:pile earth pressure coefficient is close to K^* over the whole length of the pile. A lower ratio β/χ indicates a stiffer pile: for stiffer piles the limiting moment is reached more rapidly but the displacement is of course smaller (Fig. 12).

The analysis of the displacements and moments generated in a pile by translating ground has been shown to be dependent on three dimensionless parameters giving results which can be applied generally. Defining the controlling parameters does not mean that they are all under our control. The profile of ground movement may be discoverable with appropriate instrumentation but will not be amenable to external restriction. The stiffness of the pile as a structural element is known but the detail of the interaction between the pile and ground, especially in its initial *stiffness* will be somewhat unknown and not easy to determine. The key dimensionless parameters combine structure and soil characteristics. Nevertheless, by invoking rather simple functions to describe the nature of the interaction, even these unknown quantities can be incorporated.

4. Pipeline crossed by fault

The final problem to be investigated is actually a simpler version of the previous one. We imagine a

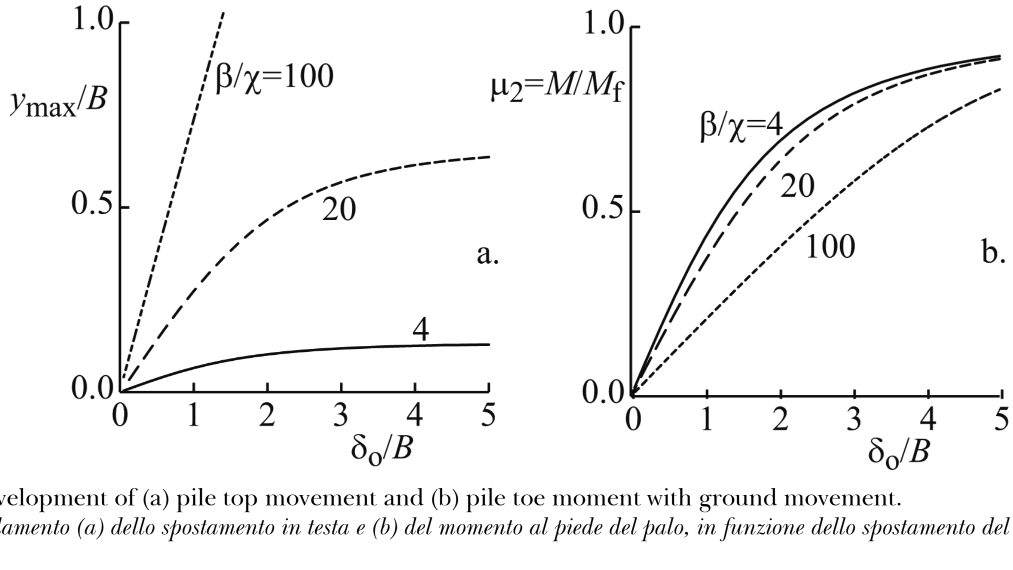

Fig. 12 – Development of (a) pile top movement and (b) pile toe moment with ground movement.
Fig. 12 – Andamento (a) dello spostamento in testa e (b) del momento al piede del palo, in funzione dello spostamento del terreno.

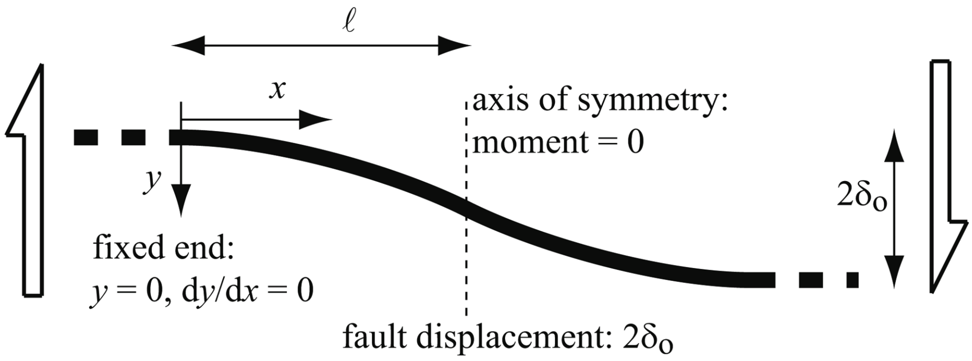

Fig. 13 – Pipeline crossing displacing fault.
Fig. 13 – Oleodotto che attraversa una faglia in movimento.

long buried flexible structure such as a pipeline or tunnel which is crossed by a fault. The fault displaces by a distance $2\delta_o$ and we want to discover the structural consequences for the pipeline. The problem is illustrated in Figure 13. Symmetry dictates that, at the centre of the pipeline, the pipeline displacement is equal to half the overall ground displacement, and that the point of inflexion in the displacement profile along the structural element implies that the moment is zero. We might suppose that the interaction between the pipeline and the ground will be independent of position along the pipeline. This simplifies somewhat the formulation of the problem and modifies the boundary conditions. However, the complicating factor is the uncertainty concern-ing the length l of the pipeline which needs to be analysed. The pile in the previous section was embedded at its lower end into a competent rock layer, and its length was thus clearly defined. The pipeline is assumed to extend to infinity in both directions. Assuming fixity at the end $x = 0$ is equivalent to proposing that the length l is sufficiently large that there is no influence of the fault displacement at this position. For the laterally loaded pile in elastic soil we observed (Fig. 2) that this might imply a length $kBl^4/4EI > 6^4 = 1296$, defined in terms of the ratio of structural stiffness and stiffness of ground-structure interaction. We will expect to discover some equivalent controlling grouping for the nonlinear problem.

The pipeline is of length l, and flexural rigidity EI, and is fixed at $x = 0$, so that both rotation and displacement are prevented (Fig. 13). A dimensionless coordinate $\eta = x/l$ defines position, with $\eta = 0$ at the fixed end and $\eta = 1$ at the point of intersection with the fault. At $\eta = 1$ the moment is zero but there is a shear force in the pipeline. The only lateral loading is provided by the moving ground. The overall fault displacement is $2\delta_o$. The displacement y of the pipeline is normalised with this displacement, $\zeta = y/\delta_o$ with the requirement that at $\eta = 1$, $\zeta = 1$.

We assume again a tanh function (Fig. 6) to describe the load generation through relative movement of ground and structural element in terms of mobilisation of earth pressure coefficient K with relative movement Δ normalised with some effective section dimension B of the structural element.

$$\frac{K}{K^*} = \tanh\left[\lambda\left(\frac{\Delta}{B}\right)\right] = \tanh\left[-\beta\zeta\right] \qquad (19)$$

As before $\beta = \lambda\delta_o/B = (\ln 3/2)\ (\delta_o/\ \Delta_{50})$ and λ and K^* are subgrade reaction parameters. The parameter K^* describes the asymptotic earth pressure coefficient reached as the relative movement increases; and K describes the presently mobilised earth pressure coefficient so that the pressure exerted on the structural element is:

$$\sigma_h = K\gamma D \qquad (20)$$

where γ is an appropriate unit weight for the soil and D is the depth of the pipeline. The limiting value K^* is a passive pressure coefficient. For a pipeline through cohesive soil with undrained strength c_u, RANDOLPH and HOULSBY [1984] suggest that $\sigma_h/c_u \cup 10$ for flow round a cylindrical pile and this would be a useful starting point in estimation of K^*.

Deformation of the pipeline is governed by the dimensionless equation

$$\frac{d^4\zeta}{d\eta^4} = \frac{1}{\chi}\frac{K}{K^*} = \frac{1}{\chi}\tanh\left[-\beta\zeta\right] \qquad (21)$$

with

$$\chi = \frac{EI\delta_o}{l^2} \frac{1}{K^*\gamma l^2 DB} \quad (22)$$

The problem is thus controlled by two parameters: β controls the initial stiffness of ground-structure interaction; and, as before, χ is the ratio of two moments, one a structural property and the other a loading characteristic. For a cantilever of length l and flexural rigidity EI subjected to a tip displacement δ_o with no other loading, the root moment is $M_r = 3EI\delta_o/l^2$. For a cantilever of length l subjected to lateral pressures over a width B given by the limiting value of the lateral stress coefficient $K = K^*$ over the entire length, the root moment is $M_f = K^*\gamma l^2 DB/2$. Thus $\chi = M_r/6M_f$. (The difference from the pile in the previous section is that the loading generated between the ground and the structure is independent of position along the structure.)

The boundary conditions for the analysis (Fig. 13) are zero deflection and slope at $x = 0: \zeta = d\zeta/d\eta = 0$ for $\eta = 0$; unit normalised displacement and zero moment at $x = l$: $\zeta = 1$ and $d^2\zeta/d\eta^2 = 0$ for $\eta = 1$.

Once the deformation Equation (21) has been solved to give a profile of normalised displacement ζ with normalised position η, the shear force and moment can be found by differentiation. The moment M is conveniently normalised with M_f:

$$\mu_3 = \frac{M}{M_f} = 2\chi \frac{d^2\zeta}{d\eta^2} \quad (23)$$

and, similarly, the shear force F is conveniently normalised with the loading F_f equivalent to M_f : the limiting earth pressure coefficient K^* mobilised along the entire length l: $F_f = K^*\gamma DBl$:

$$\Phi = \frac{F}{F_f} = \chi \frac{d^2\zeta}{d\eta^2} \quad (24)$$

Results are presented in Figures 14-17. The general form of the variation of displacement, moment and shear force along the structural element is shown in Figure 14. As before, the two controlling variables β and χ both incorporate the fault displacement δ_o. The ratio β/χ is characteristic of a specific configuration: the higher the value of β/χ the more *relatively* flexible is the pipeline or tunnel. The maximum values of dimensionless moment μ_3 and dimensionless shear force Φ are shown in Figure 15 as a function of β/χ. The variation of these maximum values is shown both in linear and log:log plots: the linear relationship for the moment in Figure 15b indicates an inverse power law relationship:

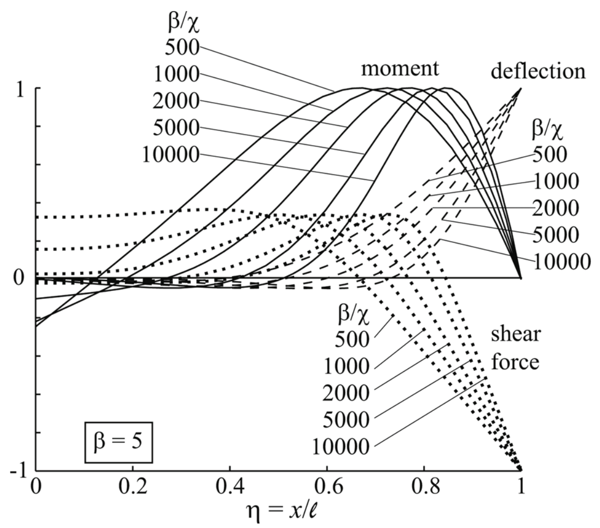

Fig. 14 – Deflection, shear force and bending moment normalised with maximum values for $\beta = 5$ or $2\delta_o/\Delta_{50} = 20/\ln 3$.
Fig. 14 – Spostamenti, sforzi di taglio e momenti, normalizzati rispetto ai corrispondenti valori massimi, per $\beta = 5$ o $2\delta_o/\Delta_{50} = 20/ln(3)$.

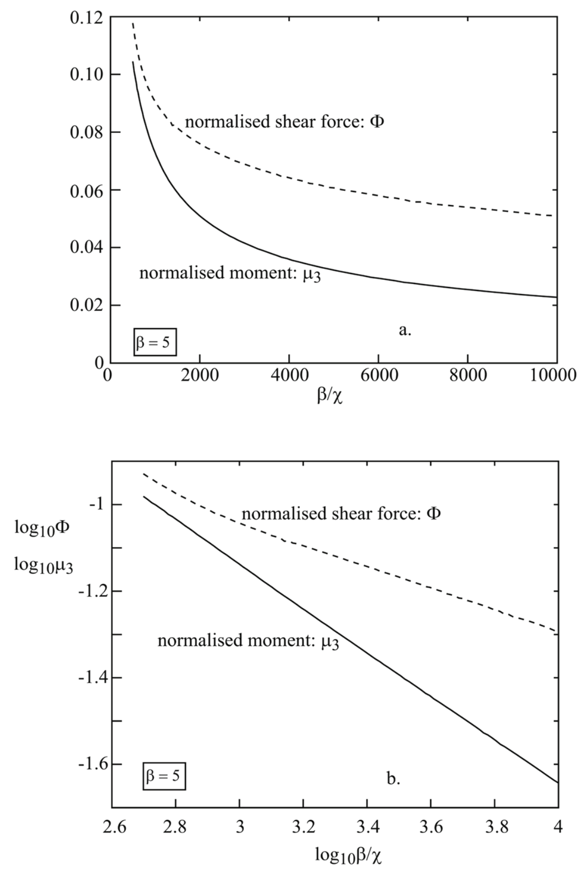

Fig. 15 – Maximum normalised shear force Φ (24) and bending moment μ_3 (23) as function of β/χ for $\beta = 5$: plotted with (a) linear scales; (b) log:log scales.
Fig. 15 – Valori massimi dello sforzo di taglio normalizzato Φ (24) e del momento normalizzato μ_3 (23) in funzione del rapporto β/χ, per $\beta = 5$, diagrammati: (a) in scala lineare; (b) in scala bilogaritmica.

$\mu_3 = (\beta/\chi)^\kappa$ where $\kappa \cup 0.5$. The shear force is not quite so simply related.

The transition from 'finite' length response to 'infinite' length response as the ratio β/χ increases from 500 to 10000 is shown in Figure 14. For a fixed value of $\beta = 5$ the distributions of pipeline deflection, and dimensionless shear force Φ and dimensionless bending moment μ_3 are shown as a function of position along the structure. All three quantities are normalised by their *maximum* values so that it is only the pattern that is being revealed. (The maximum values of shear force and moment are shown in Figure 15; the maximum value of the normalised deflection comes at the point of inflection, axis of symmetry of the structure (Fig. 13) at $\eta = 1$ and is, by definition, unity. The maximum value of shear force also occurs at $\eta = 1$ where the fault crosses the pipeline.) For each group of curves the effect of the fault displacement moves further down the pipeline towards $\eta = x/l = 0$ as β/χ decreases. The peak moment moves from about $\eta = 0.8$ to $\eta = 0.6$ as β/χ falls from 10000 to 500. More to the point, as β/χ decreases, the moment and shear force at the end of the pipeline $\eta = 0$ become significantly non-zero (Fig. 16): only for values of $\beta/\chi > 5000$ are the values of moment and shear force at $\eta = \chi/l = 0$ becoming negligible. The analysis is thus only valid in this range unless there is some actual structural fixity at some known point along the pipeline.

For the laterally loaded pile in elastic ground that we considered in the first example, visual inspection of the response suggested that the limit of 'infinite' pile response might come for values of the parametric group $kBl^4/4EI > 6^4 = 1296$. For the present analysis we have proposed that $\beta/\chi > 5000$ or

$$\frac{\beta}{\chi} = \frac{\ln 3}{2} \frac{K^* \gamma DB}{\Delta_{50}} \frac{l^4}{EI} > 5000 \qquad (25)$$

Δ_{50} is the relative displacement required to bring the interaction pressure to half the limiting value, so that an equivalence can be drawn between the pile-soil subgrade reaction coefficient k for the elastic analysis and $K^*\gamma D/2\Delta_{50}$ in the present nonlinear case. The two criteria for defining what constitutes 'infinite' length thus agree very closely ($4 \times 1296 \cup 5000$).

For acceptably large values of β/χ, which means acceptably large values of the length l for analysis, the mobilisation of shear force at the intersection of pipeline or tunnel with the displacing fault and the mobilisation of the maximum moment in the structure (Fig. 14) are shown in Figure 17 as a function of normalised fault displacement $2\delta_o/\Delta_{50}$. For each structural resultant a group of curves is shown for values of β/χ=20000, 15000, 10000 and 5000. As the value of β/χ decreases (the relative stiffness of the structural element increases) the values of the normalised moment and shear force increase for a given fault displacement. (Recall that the pipeline is analysed as a cantilever built in at $x = 0$: an equivalent analysis could be performed with the pipeline completely free at $x = 0$ analogous to the laterally loaded pile of finite length in Figure 3.)

5. Conclusions

There are various different conclusions that can be drawn from this work. The nature of the problems analysed is not expecially complex and the solution procedures are rather straightforward. How-

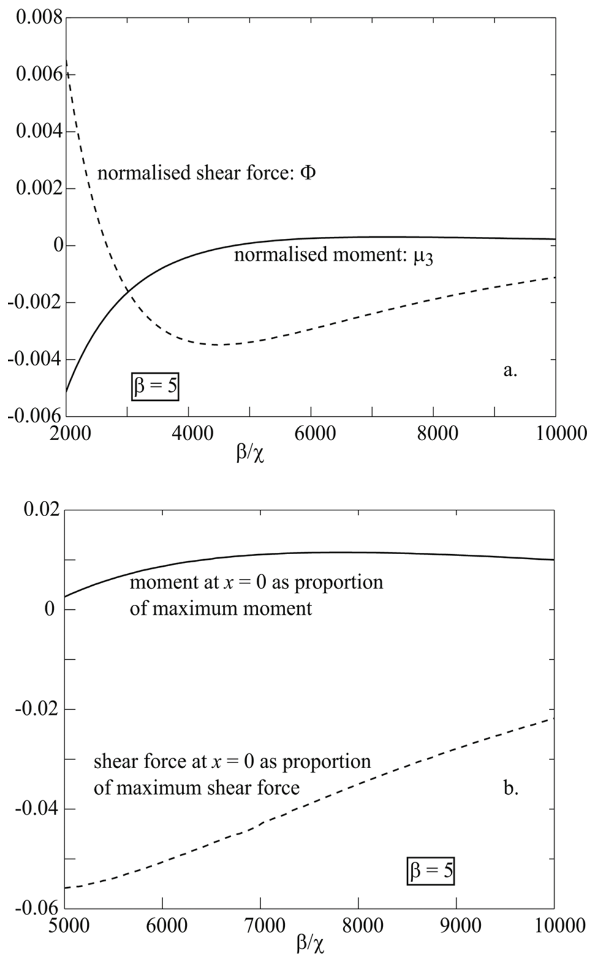

Fig. 16 – Residual shear force and moment at $\eta = x = 0$ as function of β/χ for β=5: (a) normalised shear force Φ (24) and bending moment μ_3(23); (b) residual values as proportion of maximum values.

Fig. 16 – Sforzo di taglio e momento residui nella sezione ad $\eta = x = 0$, in funzione del rapporto β/χ, per $\beta = 5$: (a) sforzo di taglio normalizzato Φ (24) e momento normalizzato μ_3 (23); (b) valori residui espressi in proporzione ai corrispondenti valori massimi.

ever, the second and third examples illustrate how one might introduce plausible nonlinear models where the data of actual behaviour are unavailable and nevertheless present results in terms of clearly defined dimensionless groups – the use of which evidently increases the applicability. Parametric study then becomes efficient.

The approach to these problems bears a close relationship to the concept of 'macroele-ment' modelling [NOVA and MONTRASIO, 1991; MUIR WOOD, 2004]. CREMER *et al.* [2001] write in the context of development of a macroelement to describe the behaviour of a footing under combined vertical, horizontal and moment loading: *'It is well known that an alternative model of the foundation behaviour obtained by the finite element method, with suitable nonlinear constitutive laws and special contact elements, requires a high degree of modelling competence and is time consuming. The macroelement provides a practical and efficient tool, which may replace efficiently, in a first approach, a costly finite element soil model, and which ensures the accurate integration of the effect of soil-structure interaction.* ' Similar sentiments may be expressed here. The nonlinear Winkler spring approach adopted blatantly disregards the continuity of the soil surrounding the pile or other structural element but introduces sufficient allusion to actual soil behaviour to allow one to understand and interpret aspects of the overall system response.

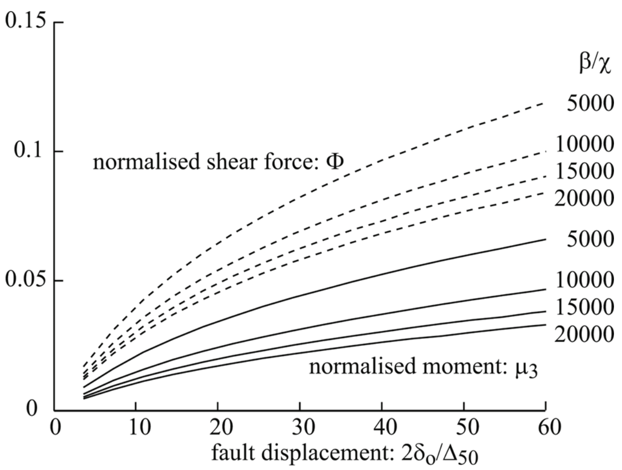

Fig. 17 – Development of shear force and moment as fault displaces: maximum normalised moment $\mu_3=2M/K^*\gamma l^2 DB$ (23); maximum normalised shear force $\Phi=F/K^*\gamma DBl$ (24).
Fig. 17 – Evoluzione dello sforzo di taglio e del momento in funzione dello spostamento della faglia: momento normalizzato massimo $\mu_3=2M/K^*\gamma l^2 DB$ *(23); sforzo di taglio normalizzato massimo* $\Phi=F/K^*\gamma DBl$ *(24).*

One of the important possibilities of analyses of the sort described here is the pedagogic potential to educate engineers in the reality of soil-structure interaction. We all know that it is important to consider the soil and structural elements as a *system* and that it is usually not possible to produce sensible results unless we do so. The use of dimensionless parameters for analysis – whether of the linear or the nonlinear problem – provides a basis for stating whether the effects of interaction are in fact important and, more especially, emphasises that the response is controlled by the *relative* stiffnesses of the ground and structure. (A salutary case history of development of stresses in a flexible integral bridge abutment is outlined by MUIR WOOD and NASH, 2000).

If practising engineers are not as aware as they should be of the importance of soil-structure interaction then we educators must take some of the blame. We atomise the teaching from the moment that the students arrive, with separate courses on structural mechanics, fluid mechanics, soil mechanics and (in my experience) retain and deepen these divisions as the degree programmes proceed with only limited time being allocated to itegrative activities which show the consequences of taking a broader system approach. Simple analytical methods such as those presented here can help the campaign to improve this awareness.

Acknowledgements

The initial analyses of the pile loaded by moving landfill were inspired by a problem posed during a Royal Society Industry Fellowship with Babtie Group. The systematic analysis of this problem was achieved while the first author was a visiting Fellow at the Institute for Industrial Science, Tokyo University, generously supported by the Foundation for the Promotion of Industrial Science. The encouragement of Professor Kazuo Konagai of the Institute for Industrial Science is very gratefully acknowledged. The helpful comments of the reviewers gave an opportunity to improve the paper which we hope we have taken.

References

CREMER C., PECKER A., DAVENNE L. (2001) – *Cyclic macro-element for soil-structure interaction: material and geometrical nonlinearities*. International Journal for Numerical and Analytical Methods in Geomechanics, 25, (13), pp. 1257-1284.

MUIR WOOD D. (2004) – *Geotechnical modelling.* E. & F.N. Spon, 488 pp.

Muir Wood D., Nash D.F.T. (2000) – *Earth pressures on an integral bridge abutment: a numerical case study*. Soils and Foundations, 40 (6), pp. 23-38.

Nova R., Montrasio L. (1991) – *Settlements of shallow foundations on sand*. Géotechnique, 41, n. 2, pp. 243-256.

Poulos H.G. (1973) – *Analysis of piles in soil undergoing lateral movement*. Proc. ASCE, Journal of the Soil Mechanics and Foundations Division, 99, (SM5), pp. 391-406.

Randolph M.F., Houlsby G.T. (1984) – *The limiting pressure on a circular pile loaded laterally in cohesive soil*. Géotechnique, 34, n. 4, pp. 613-623.

Reese L.C., Matlock H. (1956) – *Non-dimensional solutions for laterally loaded piles with soil modulus assumed proportional to depth*. Proc. 8th Texas Conf. on Soil Mechanics and Foundations, 41 pp.

Interaction of flexible structures with moving ground

Sommario

Nel presente lavoro sono descritti tre esempi di interazione tra elementi strutturali flessibili e terreno circostante in movimento. Una soluzione in forma chiusa può essere ottenuta per il problema di interazione tra un palo immerso in un terreno elastico e caricato lateralmente in testa. Quando si ritenga che la nonlinearità della risposta del sistema terreno-struttura sia importante, è necessario adottare opportune funzioni per descrivere l'evoluzione delle sollecitazioni nel palo. Anche in campo non lineare il problema può essere formulato in termini di un numero limitato di gruppi adimensionali. Un terzo caso riguarda una struttura orizzontale – un oleodotto o una galleria – attraversata da una faglia. L'individuazione delle caratteristiche del materiale che controllano la risposta del sistema per mezzo di gruppi adimensionali rende più generale l'applicazione dei risultati.