Three-dimensional periodic model for the simulation of vibrations induced by high speed trains

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Summary

A three-dimensional model for the soil-railway track system, based on a geometrical periodic formulation and taking into account the dynamic soil-track interaction, is presented. Dynamic effects induced by high speed trains on the railway track and on the soil are studied. The proposed method is then tested for a severe case in which the dynamic response of the soil-railway track system is highly amplified as the train velocity approaches a critical value.

Keywords: Floquet periodicity, soil-structure interaction, railway track, high speed train, dynamic effects.

Introduction

Vibrations induced by train traffic have an important impact on the human comfort and on the built environment. To study and then to reduce these disturbances, efficient numerical prediction tools have to be developed. Generally, two-dimensional models of the soil-railway track are proposed. However, for very long structures resting on soil (such as railway tracks), the soil-structure dynamic interaction is of great importance, and in order to take into account this phenomenon, three-dimensional models of the structure and the surrounding soil are required. Unfortunately, such analyses can be very difficult to perform even using advanced numerical methods.

To overcome this difficulty, we propose to take advantage of the geometrical periodicity of the soil-railway track system (see Sections 2, 3). In this manner, the ana-lysis for the overall soil-track system can be restricted to a reference cell. Furthermore, the generic problem can be solved using a standard subdomain approach coupling finite element models for the railway track and boundary element models for the soil (Sections 4, 5).

In this paper, attention has been paid especially to dynamic effects induced by high speed trains. Indeed, the experience in the recent years has demonstrated that high speed trains have caused numerous problems at certain sections (such as important track settlement) and have burdened railway companies with expensive maintenance works. These problems, which did not normally occur in the case of low speed train to such an extent, are caused by the dynamic effects of high speed trains on the track. Moreover, in certain rare situations, these effects can affect the safety of the train, such as the Swedish site described in Sections 6, 7. For this drastic case where the shear-wave velocities of the soil layers are very low, large amplitudes appear in the dynamic response of the soil-railway track system as the train speed approaches a critical value. This example illustrates the fact that efficient numerical models are needed to study the dynamic effects induced by high speed trains and to simulate some extreme cases that could not be tested in reality.

In a first part, the three-dimensional soil-railway track model is presented. The periodic model is described and the subdomain method used to solve the dynamic soil-railway track interaction problem is briefly recalled. The second part is devoted to the validation of the proposed method. The case studied deals with the Swedish site where the railway track, resting on a soft soil, is subjected to a moving load modelling a high speed train passing at a speed of 200 km/h (which corresponds to a critical value at which the response of the system is highly amplified).

1. Soil-railway track model

In this section, the three-dimensional model for the soil-railway track system is presented. This model is based on a geometrical periodic formulation and takes into account the dynamic soil-track interaction.

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A. Periodic model

The results from Sections 2, 3, which correspond to the extension of Floquet results (Sect. 1) to the three-dimensional soil-structure interaction problem, are used here for the soil-railway track system. The problem for the overall domain is then replaced by a generic problem in a reference cell. The next paragraph is devoted to the definition of this generic problem.

Generic problem of the three-dimensional soil-railway track system

The railway track is modelled as an unbounded open set with elastic properties resting on a stratified visco-elastic half-space. This three-dimensional domain, denoted Ω, is assumed to be periodic in the direction $\mathbf{e_y}$ (see Figs. 1a & 1b in which only the railway track structure is represented), ($\mathbf{e_x}$,$\mathbf{e_y}$,$\mathbf{e_z}$) being a Cartesian reference system.

Consequently, the problem defined in Ω can be restricted in a generic cell Ω' defined by Ω' = {**x** Ω ℑ 0 < $\mathbf{x}.\mathbf{e_y}$ < L} with **x** = (x,y,z). This generic domain is such as Ω' = $\Omega'_t \approx \Omega'_s$ where Ω'_t corresponds to the bounded cell related to the railway track and Ω'_s is the unbounded soil domain in this generic cell (see Fig. 2).

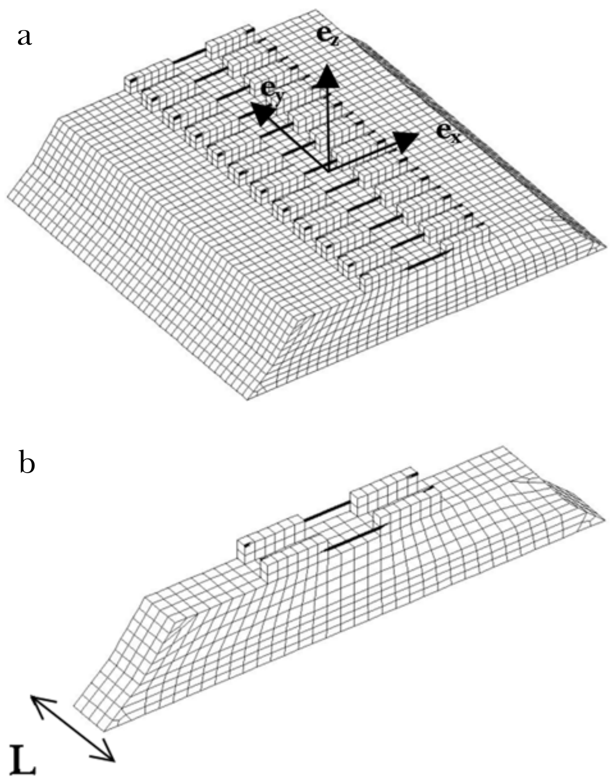

Fig. 1a – Part of the railway track structure.
Fig. 1a – Schema della struttura binari-traversine-rilevato del tracciato ferroviario.
Fig. 1b – Generic track cell.
Fig. 1b – Generica cella del tracciato ferroviario.

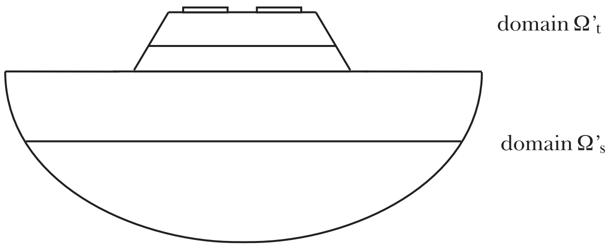

Fig. 2 – Generic cell Ω'.
Fig. 2 – Cella generica Ω'.

The boundary ≠Ω' of the generic cell Ω' can be decomposed as ≠Ω' = $\Gamma'_f \approx \Gamma' \approx \Sigma_0 \approx \Sigma_L$, with Γ'_f the part where the Neumann boundary conditions are specified, Σ_0 and Σ_L are the boundaries defined by Σ_0 = {**x** Ω ℑ $\mathbf{x}.\mathbf{e_y}$ = 0} and Σ_L= {**x** Ω ℑ $\mathbf{x}.\mathbf{e_y}$ = L}. Sommeferld's radiation conditions hold on Γ'. Finally, the position vector in the reference cell will be denoted **x**'.

Using these notations, the following generic problem has to be solved : for every wavenumber κ]–π/L, π/L[and for every circular frequency ω in the band of analysis, the track displacement $\mathbf{u}'_t(\mathbf{x}',\kappa)$ in Ω'_t and the soil displacement $\mathbf{u}'_s(\mathbf{x}',\kappa)$ in Ω'_s have to satisfy

$$\mathrm{div}\,\sigma_\beta(\mathbf{u}'_\beta) = -\rho_\beta\,\omega^2\,\mathbf{u}'_\beta \qquad \text{in } \Omega'_\beta,\ \beta\ \{t,s\}, \qquad (1)$$

$$\mathbf{t}_t(\mathbf{u}'_t) = \mathbf{f}'_t \qquad \text{on } \Gamma'_{ft}, \qquad (2)$$

$$\mathbf{t}_s(\mathbf{u}'_s) = \mathbf{0} \qquad \text{on } \Gamma'_{fs}, \qquad (3)$$

$$\mathbf{u}'_\beta(\mathbf{x}) = e^{-i\kappa L}\,\mathbf{u}'_\beta(\mathbf{x}-L\mathbf{e}_y) \qquad \text{for } \mathbf{x}\ \ \Sigma_{L\beta},\ \beta\ \{t,s\}, \qquad (4)$$

in which ρ_β (β {t,s}) is the mass density, $\sigma_\beta(\mathbf{u}_\beta)$ is the elastic stress tensor associated to the displacement field $\mathbf{u}_\beta$ and $\mathbf{t}_\beta(\mathbf{u}_\beta) = \sigma_\beta(\mathbf{u}_\beta)\,\mathbf{n}$ corresponds to the traction vector on the boundary using the outer normal convention for **n**.

Finally, the following coupling equations have to be verified

$$\mathbf{u}'_t = \mathbf{u}'_s \qquad \text{on } \Sigma', \qquad (5)$$

$$\mathbf{t}_s(\mathbf{u}'_s) + \mathbf{t}_t(\mathbf{u}'_t) = \mathbf{0} \qquad \text{on } \Sigma', \qquad (6)$$

where Σ' is the interface between the track and the soil in the generic cell.

Once the generic problem defined by Eqs. (1) to (6) has been solved, the displacement for any **x** Ω can be recovered by using the inverse Floquet transform defined by

$$\mathbf{u}_\beta(\mathbf{x}) = \frac{L}{2\pi}\int_{-\pi/L}^{\pi/L} \mathbf{u}'_\beta(\mathbf{x}',\kappa)\,e^{-i\kappa N_y L}\,d\kappa,\ \beta\ \{t,s\}, \qquad (7)$$

with $\mathbf{x} = \mathbf{x}' + N_y L\mathbf{e_y}$. In the following, the solution of the generic problem is built using a subdomain approach.

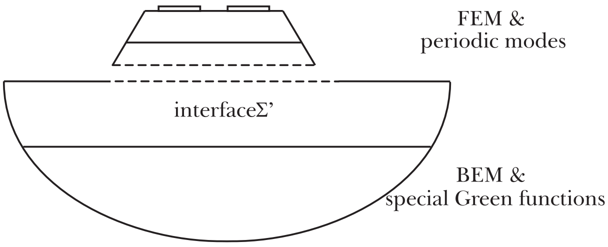

Fig. 3 – Generic cell decomposed into two subdomains.
Fig. 3 – Cella generica scomposta in due sottodomini.

B. Subdomain model

The generic soil-railway track problem is solved using a subdomain method (see Fig. 3). The three-dimensional domain considered (the generic cell) is decomposed into two subdomains (the railway track structure and the soil). Consequently, each subdomain can be independently modelled. For instance, the boundary element method (BEM) with special Green functions is used for the soil while the railway track-structure is modelled using the finite element method (FEM) and its dynamical behaviour is characterized by periodic modes.

Some results from Sections 4, 5 are briefly recalled. The displacement field $\mathbf{u}'_t$ in the bounded generic railway track cell is decomposed on a given basis of periodic modes $\{\phi_k\}_{k=1,\ldots,N}$ which satisfy Equation (4) (cf. [Sect. 2] for details concerning the construction of this kinematical basis). Then, the displacement field $\mathbf{u}'_t$ is written as

$$\mathbf{u}'_t(\mathbf{x}') = \sum_{k=1}^{N} \phi_k(\mathbf{x}')\, c_k = \boldsymbol{\phi}(\mathbf{x}')\,\mathbf{c}. \qquad (8)$$

Moreover, the soil displacement in the generic cell $\mathbf{u}'_s$ is defined by

$$\mathbf{u}'_s(\mathbf{x}') = \sum_{k=1}^{N} \mathbf{u}'_{dk}(\mathbf{x}')\, c_k, \qquad (9)$$

where $\{\mathbf{u}'_{dk}\}_{k=1,\ldots,N}$ are elastodynamic fields, which satisfy on the coupling interface Σ'

$$\mathbf{u}'_{dk} = \phi_k \quad \text{on } \Sigma'. \qquad (10)$$

Using a standard Galerkin procedure in writing the equilibrium of the generic cell in a weak sense, for any ϕ_k in the basis, the following linear system is obtained

$$\left[K_t(\kappa) - \omega^2 M_t(\kappa) + K_s(\kappa,\omega)\right]\mathbf{c}(\kappa,\omega) = \mathbf{F}_t(\kappa,\omega), \quad (11)$$

for any circular frequency ω and any wavenumber κ $]-\pi/L, \pi/L[$. Matrices $[K_t]$ and $[M_t]$ correspond respectively to the stiffness and mass matrices of the track, $[K_s]$ is the soil impedance, $\mathbf{F}_t$ is the generalized force vector applied on the structure (cf. [Sect. 2] for details concerning the computation of these quantities).

C. Dynamic response of the soil-railway track system to a constant moving load

A constant force $\mathbf{f}_t$ moving along the $\mathbf{e_y}$ direction at a constant velocity V is applied on the global railway track structure. This force is defined by

$$\mathbf{f}_t(\mathbf{x}_0,t) = F_0\, \delta(x_0-X)\, \delta(y_0-Y-Vt)\, \delta(z_0-Z)\, \mathbf{e_z}, \quad (12)$$

in which $\mathbf{x}_0=(x_0,y_0,z_0)$ is a point of Ω and where $(X,Y,Z) = \mathbf{X}$ corresponds to the position of the moving load at time t = 0. The parameter F_0 is a real constant.

It is proved (Sect. 8) that the displacement response in the frequency domain $\mathbf{u}(\mathbf{x}_1,\mathbf{X},\omega)$, for any $\mathbf{x}_1$ Ω, due to the moving load $\mathbf{f}_t(\mathbf{x}_0,t)$ defined by Eq. (12), is given by

$$\mathbf{u}(\mathbf{x}_1,\mathbf{X},\omega) = e^{i\kappa_0 Y}\, \frac{F_0}{2\pi} \int_0^L e^{-ik_0 y'}\, \mathbf{h}'(\mathbf{x}_1, X\mathbf{e_x}+y'\mathbf{e_y}+Z\mathbf{e_z}, \kappa_0, \omega)\, dy', \qquad (13)$$

in which $\kappa_0 = \omega/V$. For any κ $]-\pi/L, \pi/L[$ and circular frequency ω in the band of analysis, $\mathbf{h}'(\mathbf{x}_1, X\mathbf{e_x}+y'\mathbf{e_y}+Z\mathbf{e_z}, K, \omega)$ corresponds to the solution (at the point $\mathbf{x}_1$ Ω) of the problem defined by Eqs. (1) to (6) in which the railway track structure is subjected to a force $\mathbf{f}'_t(\mathbf{x}) = \delta(x-X)\,\delta(y-y')\,\delta(z-Z)$. It can be seen in Eq. (13) that the displacement is directly deduced from the function $\mathbf{h}'$, without computing the inverse Floquet transform defined by Equation (7).

2. Numerical modelling and validation

In order to obtain a first validation of the proposed model, the results from instrumented test runs with a high speed train on a soft soil in Sweden (cf. Sections 6 and 7) are used. It is a problematic site since measurements show large amplifications in the dynamic soil-railway track response as the train speed approaches a critical value (around 200 km/h).

In a first part, the numerical models of the railway track and the soil in the generic cell are presented. Then, convergence analyses are carried out to update the parameters of the periodic model. Finally, dynamic responses are presented and are compared with measurements.

A. Railway track model in the generic cell

The generic railway track cell is composed of two layers (see Fig. 4). The first one, with a thickness of 0.67m, is constituted of ballast whereas the second is made up of sand and gravels. On this generic track-structure, four sleepers in concrete are taken into account and are connected in pairs by means of steely crossbars. Moreover, rails have been

Tab. I – Mechanical characteristics of the track.
Tab. I – Caratteristiche meccaniche del sottofondo ferroviario.

	ballast	sub-layer	crossbar	sleeper	rail	pad
E (N/m^2)	2.9310^8	2.9310^8	$2.1\ 10^{11}$	$3\ 10^{10}$	$2.1\ 10^{11}$	/
ρ (kg/m^3)	1800	1800	7800	2200	7800	/
ν	0.3	0.3	0.3	0.25	0.3	/
k (N/m)	/	/	/	/	/	$159\ 10^6$

E: Young's modulus, ρ: mass density, ν: Poisson's ratio, k: vertical stiffness.

Tab. II – Mechanical and geometrical characteristics of the soil.
Tab. II – Caratteristiche meccaniche e geometriche del terreno.

	crust	organic clay	clay	clay	half space
C_P (*m/s*)	500	500	1500	1500	1500
C_s (*m/s*)	65	33	60	85	100
ρ (*kg/m^3*)	1500	1260	1475	1475	1475
β	0.063	0.058	0.098	0.064	0.060
h (m)	1.1	3.0	4.5	6.0	/

C_P : pressure wave velocity, C_s : shear wave velocity, ρ: mass density, β: hysteretic damping ratio, h: thickness.

modelled and are connected to the sleepers with pads. The geometrical and mechanical characteristics are given respectively in Figure 4 and Table I.

A hysteretic damping of about 2% is taken into account.

B. Soil model in the generic cell

The soil is assumed to be a stratified visco-elastic half-space. It consists of a weathered clay crust and a layer of extremely soft organic clay over soft marine clays. The mechanical and geometrical characteristics are given in Table II (it can be seen that shear-wave velocities are very low).

C. Convergence analyses

In order to get a significant response in the soil for the frequency range B=2π∞[1,80] rad/s, a convergence analysis with respect to the number N_y of cells has been carried out. For this, a norm associated to the soil impedance $[K_s]$ is introduced and is defined by

$$(|||[K_s]|||_B)^2 = \int_B |||[K_s(\omega)]|||^2 d\omega \text{ with}$$

$$|||[K_s(\omega)]|||^2 = Tr([K_s(\omega)][K_s(\omega)]^*) \forall\omega\ B, \quad (14)$$

in which Tr denotes the trace of matrices and where the subscript * corresponds to the adjoint of matrices. On Figure 5, one can see that a good convergence is obtained for N_y=70 cells.

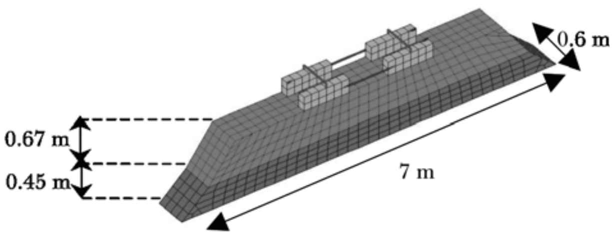

Fig. 4 – Geometrical characteristics of the generic track cell.
Fig. 4 – Caratteristiche geometriche della generica cella del tracciato ferroviario.

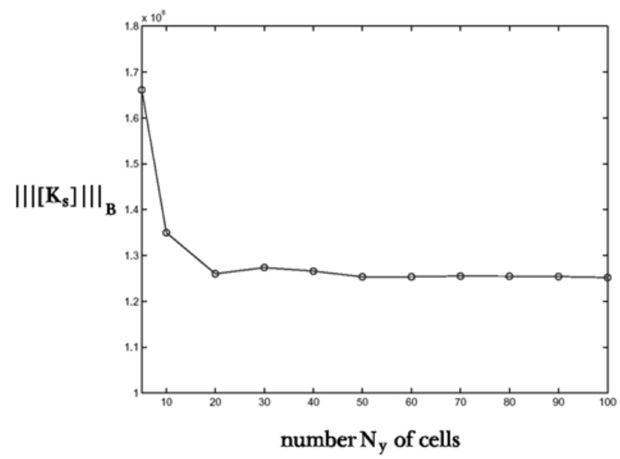

Fig. 5 – Convergence with respect to the number of cell.
Fig. 5 – Convergenza della simulazione numerica rispetto al numero di celle N_y.

For the generic track cell, a convergence analysis with respect to the number N of periodic modes (see Eq. 8) has been performed for the band B defined previously. For this, a norm associated to the complex-valued displacement vector $U = (U_1, U_2, ..., U_n)$ in the generic track-structure is introduced and is defined by

$$(||U||_B)^2 = \int_B ||U(\omega)||^2 d\omega \text{ with}$$

$$|U(\omega)||^2 = \sum_{i=1}^{n} |U_i(\omega)|^2 \ \forall\omega \ B. \qquad (15)$$

A number of N=15 modes has been considered. Moreover, an optimal interval for the wavenumber parameter has be chosen.

D. Dynamic response of the soil-railway track system to a constant moving load

Since the parameters of the periodic model have been updated, the dynamic response in the soil-railway track system under a moving load can be computed. In order to get a good understanding of the dynamic behaviour of the system, a suitable visualization model is built in which the generic cell is repeated 11 times in the $\mathbf{e_y}$ direction (see Fig. 6). The middle cell corresponds to the reference cell.

It is worth to note that this visualization model, by construction, is "bounded" whereas the solution can be calculated as far as it would be desired.

The soil-railway track system is subjected to a moving load which models a train whose speed equals 200 km/h. Only one axle has been modelled (with an axle load of 360 kN). The dynamic responses are first calculated for the generic cell, using the results from Section I. Then, the dynamic responses for the overall system are deduced by using Equation (7).

Figure 7 shows the dynamic behaviour of the soil-railway track system when the moving load acts on the middle cell (at time t ≅ 5.06s; the time scale is shown on Figure 8). Only the vertical displacement component is represented. It can be seen that the visualization model offers an interesting view for the comprehension of the wave propagation phenomenon in the soil and along the track.

And, when responses of the railway track and of the soil are animated (Fig. 7 has been extracted from this animation), the dynamic effect of the high speed train is conspicuous. Indeed, amplified vibrations of the railway track structure can be seen before the passage of the train together with the vibration tail generated behind the train. These phenomena can also be observed on Figure 8. Moreover, on this figure, simulations (blue line) are compared with measurements presented in Section 7 (red cross) for the vertical displacement at the position (0,0.3,0).

A good agreement between the calculated and the measured displacement is achieved.

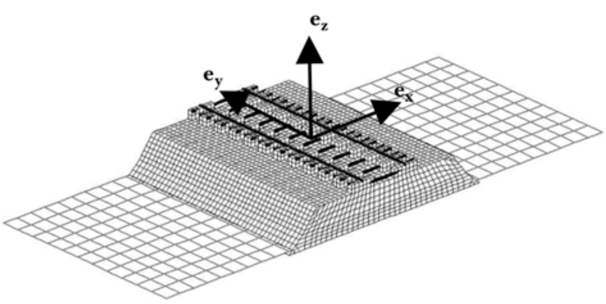

Fig. 6 – Global soil-railway track model.
Fig. 6 – Modello globale del tracciato ferroviario.

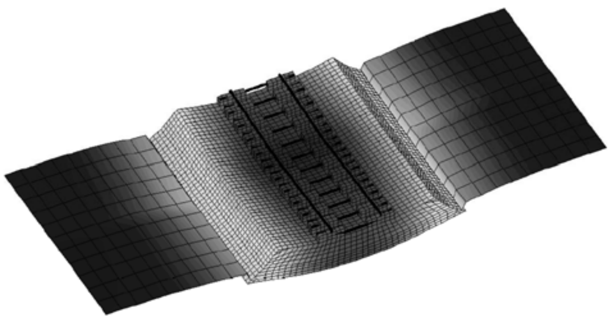

Fig. 7 – Soil-railway track dynamic response to a moving load, at t ≅ 5.06s.
Fig. 7 – Risposta dinamica del sistema terreno-binari alle sollecitazioni indotte da un carico viaggiante, al tempo t ≅ 5.06s.

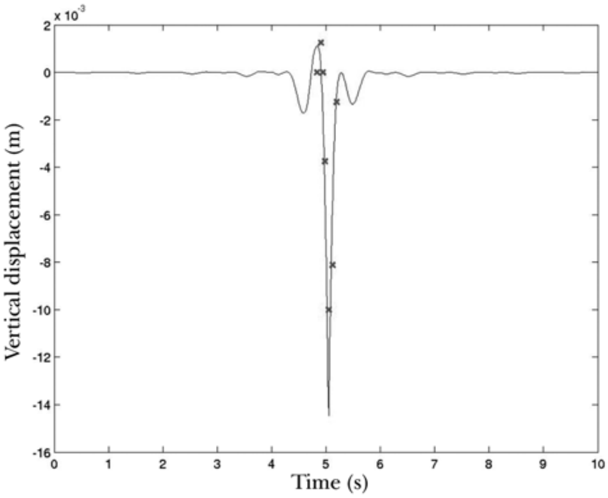

Fig. 8 – Comparison between simulation (blue line) and measurement (red cross) for the vertical displacement at (0,0.3,0).
Fig. 8 – Confronto fra dati misurati (croci rosse) e simulazione numerica (linea continua blu) relativi agli spostamenti verticali per un punto di coordinate (0,0.3,0).

3. Conclusions and perspectives

A linear model of the soil-railway track system, based on a geometrical periodic formulation and taking into account the dynamic soil-track interaction, is proposed. With such a model, a detailed three-dimensional description of the railway track and the soil can be used.

Parameters of the periodic model have been defined using convergence analyses. Then, the dynamic responses of the soil-railway track system, subjected to a moving load which models a high speed train, have been computed. The model has been tested for a severe case in which amplification in response amplitude of the soil-track system is observed. A good correlation with the measurements is shown, highlighting the ability of the proposed approach to describe dynamic effects induced by high speed trains.

Moreover, in order to get a complete validation of the method, it would be interesting to model all the train's axle loads and to study the modification in the dynamic response.

Finally, another undesirable effect associated to the high speed train is the stronger degradation of the track (which is observed after a few years even though the axle load for high speed train is lower than that of freight trains). Consequently, numerical models have to be developed in order to simulate the long-term non-linear behaviour of the track. The proposed approach can then represent the first building block of a general model for the simulation of the non-linear soil-structure responses due to repeated loads (in Sect. 9, this methodology has been already used).

Acknowledgements

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Modello periodico tridimensionale per la simulazione delle vibrazioni indotte da treni ad alta velocità

Sommario

L'articolo illustra un modello tridimensionale dell'insieme binario-sottostruttura basato su una formulazione geometrica periodica in grado di tener conto dell'interazione dinamica suolo-struttura. Sono stati studiati gli effetti dinamici indotti dal passaggio di treni ad alta velocità sui binari ferroviari e sul terreno sottostante. L'approccio proposto è stato quindi applicato ad un caso critico in cui la risposta dinamica del sistema terreno-binari è largamente amplificata in quanto la velocità del convoglio raggiunge un valore critico.

Parole chiave*: Periodicità di Floquet, interazione terreno-struttura, tracciato ferroviario, treni ad alta velocità, effetti dinamici.*