Cyclic properties of Toyoura Sand at small to medium strains in simple shear test

Giuseppe Lanzo* Beniamino D’Elia*

Summary
Cyclic properties of Toyoura sand were investigated at small to medium shear strains using a simple shear apparatus for small-strain testing. The cyclic properties are expressed in terms of the maximum (\(G_{\text{max}}\)), secant (\(G_s\)) and normalized shear moduli (\(G_s/G_{\text{max}}\)), and the damping ratio (\(D\)). Cyclic shear strain amplitude (\(\gamma_c\)), vertical effective consolidation stress (\(\sigma_{vc}\)), frequency of cyclic loading (\(f\)) and number of cycles (\(N\)) were varied among the tests. The objective of the paper is twofold: (a) to systematically analyze and present the results of simple shear testing on Toyoura sand with attention paid to the effects of \(\gamma_c\), \(\sigma_{vc}\), \(f\), and \(N\) on shear modulus and damping ratio; (b) to compare the simple shear test results with the laboratory results obtained by other researchers using different testing techniques.

1. Introduction
The importance of cyclic properties of soils at small to medium strains has been widely recognized in many geotechnical engineering problems for the last forty years. Analysis of an engineering problem that involves cyclic loading of soils requires the determination of two main parameters, namely the secant shear modulus, \(G_s\), and the damping ratio, \(D\). These parameters can be obtained from measurements of stress-strain loops, which describe the soil response to cyclic loading. An idealized cyclic stress-strain loop is sketched in Fig. 1a. In this figure, \(\tau =\) shear stress, \(y =\) shear strain, \(\tau_c =\) cyclic shear stress amplitude, \(y_c =\) cyclic shear strain amplitude, \(G_{\text{max}} =\) maximum shear modulus at very small strains and \(G_s =\) secant shear modulus corresponding to \(\tau_c\) and \(y_c\). The damping ratio \(D\) can be evaluated by the following expression [JACOBSEN, 1930]:

\[
D = \frac{1}{4\pi} \frac{\Delta W}{\frac{1}{2} \tau_c y_c}
\]

(1)

where \(\Delta W\) is the area enclosed by the loop. Due to the non-linearity of the stress-strain curve, both \(G_s\) and \(D\) are strong function of the cyclic shear strain amplitude, \(y_c\). As \(y_c\) increases, \(G_s\) decreases and \(D\) increases. In the current practice, it is customary to present the variations of \(G_s\) and \(D\) with \(y_c\) in terms of normalized shear modulus (\(G_s/G_{\text{max}}\) vs. \(y_c\)) and damping ratio (\(D\) vs. \(y_c\)) curves, such as sketched in Fig. 1b.

Over the past two to three decades, considerable efforts have been devoted toward accurate measurements of cyclic stress-strain loops. These efforts have resulted in significant developments in laboratory testing techniques that provided the opportunity to improve significantly the characterization of cyclic properties of soils. In particular, cyclic static apparatus (i.e., cyclic triaxial, cyclic torsional shear and cyclic simple shear) equipped with local transducers are nowadays capable of providing precise and reliable measurements of cyclic soil properties over a wide range of strains. These apparatus represent valid alternatives to dynamic tests (i.e., resonant column and bender element tests) to determine cyclic properties of soils in the small \((y_c < 0.001%)\) and medium \((0.001% \leq y_c \leq 0.1%)\) strain ranges.

In addition to that, extensive experimental studies on natural and reconstituted soils as subjected to cyclic loading have been performed in the laboratory. These studies allowed a better identification of the main factors affecting cyclic behavior of soils such as plasticity index (PI), void ratio (e), cyclic shear strain amplitude (\(y_c\)), effective mean confining pressure (\(\sigma_{vc}\)), number of cycles (\(N\)) and frequency of cyclic loading (\(f\)). The significant advances that have been made in the cyclic testing of soils at small to medium strain levels are described in a number of state-of-the-art papers and specialty conference proceedings [TATSUMI and SHIBUYA, 1992; SHIBUYA et al., 1994; STOKES et al., 1995; TATSUMI et al., 1995; TATSUMI et al., 1997; STOKES et al., 1999; JAKOBEK et al., 1999].

The purpose of this paper is to present the results of a laboratory study on the cyclic properties of Toyoura sand subjected to simple shear loading conditions. A relatively recent simple shear appara-

* Dipartimento di Ingegneria Strutturale e Geotecnica, Università di Roma "La Sapienza".
minantly quartz fine to medium sand, having a density of soil particles \( \rho_s = 2.65 \text{ Mg/m}^3 \), a mean grain size \( D_{50} = 0.2 \text{ mm} \), a coefficient of uniformity \( U_c = 1.3 \). The maximum and minimum void ratio are, respectively, \( e_{\text{max}} = 0.975 \) and \( e_{\text{min}} = 0.561 \). The grain size distribution curve is presented in Fig. 2.

3. Testing apparatus

The Double Specimen Direct Simple Shear (DSDSS) device was used for the present study. This device has been designed and built at the University of California at Los Angeles and its detailed description can be found in DOROU DAN and VUCETIC [1995]. Therefore, it will be only briefly illustrated hereafter.

The key feature of the DSDSS device is that two specimens of the same soil are tested simultaneously, instead of just one. The specimens are of the Norwegian Geotechnical Institute (NGI) type, i.e., they are contained in wire-reinforced rubber membranes that laterally confine the specimens to approximate simple shear deformations during the application of horizontal loads. A schematic configuration of the DSDSS device is shown in Fig. 3. Such a peculiar configuration, in conjunction with very stiff components of the device, has been found to adequately limit some problems such as mechanical compliance of the apparatus and frictional resistance to simple shear horizontal loading. These problems are otherwise typical of a standard NGI direct simple shear device and do not allow the investigation of cyclic properties of soils at small strains.

The DSDSS device has been originally designed to investigate the small-strains cyclic behavior of soils. However, the versatility of this device is such that it was also used for small-strain testing was used for the experimental study. This testing is unique in that simple shear tests can be conducted on a pair of specimens of the same soil over a large range in cyclic shear strain amplitude, \( \gamma_c \), and vertical effective confining pressure, \( \sigma'_{vc} \). The objective of the tests was to investigate the effects of \( \gamma_c, \sigma'_{vc}, N \) and \( f \) on \( G_{\text{max}}, G_s \) and \( D \). In addition to that, the simple shear test results are compared to those obtained on the same sand by other researchers using different laboratory testing techniques.

2. Tested soil

Standard Japanese Toyoura sand was employed as test material. This sand has been widely used for laboratory stress-strain testing in Japan since the Seventies [e.g., IWASAKI and TATSUURA, 1977; IWASAKI et al., 1978; KOKUSHO, 1980]. Toyoura sand is a predo-

![Fig. 1](image1.png)

**Fig. 1** - (a) Idealized symmetric cyclic stress-strain loop; (b) typical format of normalized shear modulus \((G/\gamma_c)\) and damping ratio \( (D) \) vs. cyclic shear strain amplitude \( \gamma_c \) curves.

![Fig. 2](image2.png)

**Fig. 2** - Grain size distribution curve of Toyoura sand.

![Fig. 3](image3.png)

**Fig. 3** - Curva granulometrica della sabbia di Toyoura.
that it is possible to measure, in a single test, cyclic properties of soils in a very wide range of strains, from very small to very large. As a matter of fact, shear moduli and damping ratios of various soils in the range of $\gamma_c$ between approximately 0.0004% and 1.0% have been successfully measured in the DSDSS device [Lanzo et al., 1997; Vucetic et al., 1998a, 1998b; Lanzo et al., 1999].

The capability of the DSDSS device to measure cyclic soil properties at small strains is illustrated in Fig. 4, which plots some representative records of a series of cyclic tests on Toyoura sand. The figure displays the stress-strain curves obtained at $\sigma_{vc} = 180$ kPa for five different shear strain amplitudes, $\gamma_c$, i.e. 0.00043%, 0.00083%, 0.0013%, 0.0033% and 0.0058%. At the smallest $\gamma_c = 0.00043\%$, the modulus $G_s$ could be determined with confidence, but damping ratio $D$ could not, because the stress-strain loop could not be clearly recorded due to the ambient vibrations and electrical interference. At somewhat larger $\gamma_c = 0.00083\%$, the cyclic loop was recorded but still not very clearly, while again $G_s$ could be determined with confidence. At $\gamma_c = 0.0013\%$ and larger, the cyclic loops were clearly recorded and both $G_s$ and $D$ could be determined with confidence. The area of the loop to be introduced in

\[ \text{Eq. (1)} \]

for the determination of damping ratio was calculated by a program developed at the University of California at Los Angeles using the mathematical processing program MATLAB. In conclusion, with the DSDSS device the $G_s$ modulus can be determined with confidence for very small $\gamma_c$ close to 0.0001%, while $D$ can be determined with confidence for $\gamma_c$ larger than approximately 0.001%.

4. Testing procedure and testing program

The tests were conducted on air-dry Toyoura sand specimens. The specimens were cylindrical,
6.6 cm in diameter by approximately 2 cm high. They were prepared by pouring dry sand with a spoon in five layers into the wire-reinforced membranes and tamping each layer with a small wooden red. The specimens were tested following the original NGI constant-volume equivalent-undrained direct simple shear testing procedure [Bjerrum and Landva, 1966]. Because the specimens are enclosed in the reinforced membranes, which are intended to keep the cross-sectional area constant, the constant volume condition reduces to the requirement that the specimen height be maintained constant. This is accomplished by firmly fixing, after the completion of consolidation, the bottom and top caps to the top and bottom plates (Fig. 3) of the device with the help of threaded rods mounted in the plates. Consequently, the variation of the vertical stress during cyclic shearing, which occurred due to the imposed constant volume conditions, was considered equivalent to the variations of the pre-nated pressure that would have developed in a truly undrained test therefore, the results presented in this paper: provide a reasonable approximation of the undrained cyclic simple shear conditions [Devito et al., 1987; Airey and Wood, 1987].

Two samples of Toyoura sand, labeled A and B, were tested in the DSDSS device according to the testing program summarized in Tab. 1. From each sample a pair of the DSDSS specimens was cyclically sheared. On sample A, a series of four cyclic tests, A1 to A4, was conducted at 4 different vertical effective consolidation stresses, σ̇ve. The values of σ̇ve and those of the corresponding void ratio at the end of primary consolidation, e, are listed in Tab. 1. In each of the 4 cyclic tests several consecutive cyclic strain-controlled stages were applied, with γc being constant in each stage and larger in each consecutive stage. The range of γc applied in each test is listed in Tab. 1, showing that the smallest γc was about 0.0003% while the largest was between 0.932% and 0.86%. The range of applied cyclic loading frequencies, f, varied approximately between 0.02 and 1.5 Hz, and is also listed in Tab. 1. The number of cycles applied in each strain-controlled stage was on average 16. The shape of cyclic strain was between triangular and sinusoidal and only for few tests a trapezoidal shape of cyclic strain was employed.

On sample B, only two staged cyclic tests, labeled B1 and B2, were conducted. The values of σ̇ve, e, γc, and f are listed in Tab. 1. Samples B1 and B2 were cyclically tested to examine the repeatability of the testing technique with the DSDSS device.

5. Repeatability of testing with the DSDSS device

The repeatability of the testing method is a very important quality assurance factor when performing an experimental investigation, especially when the testing is sensitive, such as small-strain testing, and when it is conducted with a new type of device. The repeatability of the DSDSS testing method has already been tested on clay specimens [Doboudjian and Vucetic, 1995] but it has never been tested on sand specimens. For this reason, two pairs of Toyoura sand specimens were tested under the same loading conditions and the results were compared.

As shown in Tab. 1, the specimens in tests A1 and B1 had very similar void ratio values and were tested under the same σ̇ve = 98 kPa; analogously, the specimens in tests A2 and B2 had similar void ratio and were tested under the same σ̇ve = 180 kPa. The comparisons between the Gs and D values obtained in the two tests at similar or identical levels of γc are presented in Fig. 5. The clusters of the data points pertain to different cyclic strain-controlled stages, and each data point in a cluster pertain to a single cyclic stress-strain loop recorded during the given stage. It can be seen that the Gs and D data points of tests A1 and B1, as well as these of tests A2 and B2, plot on top of each other thus confirming an excellent repeatability of testing.

Tab. 1 – Summary of the testing program and test conditions.
Tab. 1 – Quadro riassuntivo del programma e delle condizioni di prova.

<table>
<thead>
<tr>
<th>Samples name</th>
<th>Test name</th>
<th>Effective vertical consolidation stress σ̇ve (kPa)</th>
<th>Void ratio at the end of primary consolidation e</th>
<th>Range of cyclic shear strain amplitudes applied γc (%)</th>
<th>Range of frequencies applied f (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A1</td>
<td>90</td>
<td>0.594</td>
<td>0.0003-0.058</td>
<td>0.02-1.4</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>180</td>
<td>0.595</td>
<td>0.00042-0.86</td>
<td>0.02-1.0</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>420</td>
<td>0.591</td>
<td>0.00042-0.86</td>
<td>0.013-1.5</td>
</tr>
<tr>
<td></td>
<td>A4</td>
<td>900</td>
<td>0.588</td>
<td>0.00042-0.092</td>
<td>0.063-0.77</td>
</tr>
<tr>
<td>B</td>
<td>B1</td>
<td>90</td>
<td>0.590</td>
<td>0.00043-0.011</td>
<td>0.02-0.31</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>180</td>
<td>0.589</td>
<td>0.00041-0.006</td>
<td>0.07-0.8</td>
</tr>
</tbody>
</table>
6. Test results

6.1. Reduction of secant shear modulus, $G_s$, with $\gamma_c$

The data points corresponding to the values of $G_s$ and $\gamma_c$ applied in the consecutive cyclic strain-controlled stages of the tests A1 through A4 are plotted in Fig. 6. For each test, the data points in Fig. 6 are connected with solid lines to obtain the $G_s$ versus $\gamma_c$ relationship. In addition to that, to estimate the values of $G_{\text{max}}$, the $G_s$ versus $\gamma_c$ relationships are extrapolated to $\gamma_c = 0.0001\%$ by dashed lines. The secant shear modulus $G_s$ at $\gamma_c = 0.0691\%$ is commonly considered equal to the maximum shear modulus, $G_{\text{max}}$.

As expected, in Fig. 6 $G_s$ consistently decreases with $\gamma_c$ in all 4 tests. The $G_s$ values pertaining to a single test remain approximately equal to the maximum value $G_{\text{max}}$ at or below $\gamma_c = 0.001\%$, which is around the value of the elastic threshold shear strain for sands [Vucetic, 1994]. At $\gamma_c$ higher than 0.001\% the shear modulus $G_s$ decreases very rapidly as $\gamma_c$ increases.
Fig. 6 also displays the trends of $G_c$ with $\sigma_{vc}'$ for a given $\gamma_c$. The figure shows that $G_c$ consistently increases with $\sigma_{vc}'$. As $\sigma_{vc}'$ increases, the $G_c$ versus $\gamma_c$ curves plot higher. These trends are well known and have been observed in other investigations since seventies [Hardin and Drnevich, 1972a and 1972b; Iwasaki and Tatsuoka, 1977; Korisho, 1980].

6.2. Evaluation of the maximum shear modulus, $G_{max}$

Hardin and Drnevich [1972b] showed that $G_{max}$ of sands depends primarily on mean effective confining stress and void ratio. Iwasaki and Tatsuoka [1977] from the results of resonant column tests on isotropically consolidated specimens on fifteen clean sands found that $G_{max}$ could be expressed as:

$$G_{max} = 900 \left(\frac{2.17-e}{1+e}\right)^2 \sigma_m^{0.40}$$  \hspace{1cm} (2)

where $F(e) = (2.17-e)^2/(1+e)$ is a void ratio function [Hardin and Richart, 1965] and $\sigma_m$ is the mean effective stress. In Eq. (2) $G_{max}$ and $\sigma_m$ are expressed in kPa.

As stated above, the $G_{max}$ values of Toyoura sand were estimated by extrapolating the $G_c$ versus $\gamma_c$ curves in Fig. 6 to $\gamma_c = 0.0001\%$. Considering that lateral stresses in simple shear conditions are typically not measured and consequently no information on their magnitude is available, the values of $\sigma_m'$ were estimated. By using the expression $\sigma_m' = (\sigma_v' + 2\sigma_h')/3$, where $\sigma_v'$ is vertical effective consolidation stress and $\sigma_h'$ is horizontal effective consolidation stress, it was assumed that $\sigma_v' = \sigma_{vc}'$ and $\sigma_h' = K_0 \sigma_v'$, where $K_0$ is the coefficient of earth pressure at rest. To estimate $K_0$, the empirical equation proposed by Jaky [1948], which expresses $K_0$ as a function of the effective stress friction angle, $\phi'$, was employed. The effective stress friction angle for dense Toyoura sand introduced in the Jaky's expression is equal to $\phi' = 40^\circ$ [Yamashita et al., 2001] which results in $K_0 = 0.36$.

The $G_{max}$ and $\sigma_m'$ values estimated as said above are plotted in Fig. 7 as a function of $\sigma_m'$ by means of two relationships. In Fig. 7a, the values of $G_{max}$ are simply plotted against $\sigma_m'$, while in Fig. 7b they are normalized by the void ratio function $F(e)$ established for clean sands by Iwasaki and Tatsuoka [1977] and reported in Eq. (2). In both plots it is apparent the significant increase of maximum shear modulus as $\sigma_m'$ increases. Both the $G_{max}$ and $G_{max}/F(e)$ versus $\sigma_m'$ relationships can be represented quite accurately by a straight line on the log-log scale. In particular the slope of the regression line, indicated as dashed line in the $G_{max}/F(e)$ versus $\sigma_m'$ plot (Fig. 7b), is equal to 0.47. For the sake of comparison, also drawn in Fig. 7b with a solid line is the Iwasaki and Tatsuoka [1977] equation which is characterized by a slope of 0.40. It can be seen that the $G_{max}/F(e)$ values are smaller than those calculated by the Iwasaki and Tatsuoka [1977] equation, the difference reducing as $\sigma_m'$ increases. The relation $G_{max}/F(e) - \sigma_m'$ will be examined in more detail in a subsequent section, together with the numerous data from literature.
6.3. Reduction of normalized shear modulus, $G/G_{\text{max}}$ with $\gamma_c$

The reduction of $G$ with $\gamma_c$ is presented in Fig. 8 in the normalized form with respect to $G_{\text{max}}$, i.e., in a customary $G/G_{\text{max}} - \gamma_c$ plot. All four values of $\sigma'_{\text{se}}$ applied in test A are reported in the figure.

Each single data point represents an average of the cluster of the data points obtained for all cyclic stress-strain loops recorded at the given $\gamma_c$ in a given test. It can be seen in Fig. 8 that as $\sigma'_{\text{se}}$ increases the $G/G_{\text{max}} - \gamma_c$ data points consistently shift towards right, that is the $G/G_{\text{max}} - \gamma_c$ relationship of Toyoura sand is very sensitive to the vertical effective confining stress. These results are in agreement with the results by Iwasaki et al. [1978] and KURUSHI [1980] for Toyoura sand and by Ni [1987], LANDO et al. [1997] and STOKES et al. [1999] for other different sands. It is thus confirmed that for sands the effects of $\sigma'_{\text{se}}$ on the $G/G_{\text{max}} - \gamma_c$ relationship is significant.

In Fig. 8, the $G/G_{\text{max}} - \gamma_c$ curves proposed by SEED and IDRIS [1970] for sandy soils are also plotted for comparison purposes. The upper and lower bounds of the $G/G_{\text{max}} - \gamma_c$ curves are shown by dashed lines and an average $G/G_{\text{max}} - \gamma_c$ curve is shown by a solid line. It can be seen that the $G/G_{\text{max}} - \gamma_c$ data points corresponding to different $\sigma'_{\text{se}}$ values fall in a relatively narrow range which is generally above the upper bound of the Seed & Idriss range, at least for $\gamma_c \leq 0.1\%$.

6.4. Damping ratio, $D$, versus $\gamma_c$

The variation of the damping ratio, $D$, data with $\gamma_c$ is presented in Fig. 9 for all values of $\sigma'_{\text{se}}$ applied in test A and for two different values of the number of cycles, $N$, namely $N = 2$ (Fig. 9a) and $N = 7 - 10$ (Fig. 9b).

As expected, damping ratio increases at a given $\sigma'_{\text{se}}$ as $\gamma_c$ increases. Further, at a given $\gamma_c$, damping ratio decreases as $\sigma'_{\text{se}}$ increases. This effect is more pronounced at $N = 2$ than at $N = 7 - 10$. These results are in agreement with the results by TATSUOKA et al. [1978] and KURUSHI [1980] for Toyoura sand, and with the results by Ni [1987], KOM [1991], VUCETIC et al. [1998b] and STOKES et al. [1999] for other different sands. It is thus confirmed that for sands the effect of $\sigma'_{\text{se}}$ on the $D - \gamma_c$ relationship can be significant, especially if the first cycles are taken into account for comparison.

In Fig. 9, the $D - \gamma_c$ curves proposed by SEED and IDRIS [1970] for sandy soils are also plotted for comparison. The upper and lower bounds of the $D - \gamma_c$ curves are shown by dashed lines and an average $D - \gamma_c$ curve is shown by a solid line. An overall agreement can be recognized between the Seed and Idriss damping ratio curves and the $D - \gamma_c$ data obtained with the DSDDSS device. However, as for the normalized shear modulus, it is apparent that the damping data points, in the small strain range, plot below the lower bound established by SEED and IDRIS [1970].

6.5. Effect of number of cycles, $N$, on shear modulus, $G_{\text{se}}$, and damping ratio, $D$

The variations of shear modulus, $G_{\text{se}}$, and damping ratio, $D$, with the number of cycles, $N$, are shown in Fig. 10 for $\sigma'_{\text{se}} = 180$ kPa (test A2 in Tab. I). These variations are shown at three different ranges of cyclic shear strain amplitude $\gamma_c$, namely $\gamma_c = \ldots$
0.0027 - 0.0032% (Fig. 10a), \( \gamma_c = 0.012 - 0.014\% \) (Fig. 10b) and \( \gamma_c = 0.027 - 0.032\% \) (Fig. 10c). At \( \gamma_c = 0.0027 - 0.0032\% \) (Fig. 10a) no appreciable changes of \( G_s \) and \( D \) with \( N \) can be observed. This is in agreement with the results from literature which show that below the volumetric threshold shear strain, \( \gamma_v \), which for sands is around 0.01% [Vucetic, 1994], the influence of \( N \) on \( G_s \) and \( D \) is negligible [Kim, 1991; Stokoe et al., 1995; Lo Presti et al., 1997]. At \( \gamma_c = 0.012 - 0.014\% \) (Fig. 10a), which is close to the volumetric threshold shear strain, as \( N \) increases \( G_s \) remains practically constant while \( D \) decreases. It is also noted that \( N \) has a significant effect on \( D \) in the first 10 cycles, while little effect can be noted between \( N = 10 \) and \( N = 20 \). At \( \gamma_c = 0.027 - 0.032\% \) (Fig. 10c), well beyond the volumetric threshold shear strain, both \( G_s \) and \( D \) are affected by the number of cycles. It can be noted a slight increase, on average, of shear modulus as the number of cycles increases while the damping ratio drops significantly between \( N = 1.5 \) and \( N = 10 \). This means that the effect of \( N \) above \( \gamma_v \approx 0.01\% \) is much more important for \( D \) than for \( G_s \). Further, the change in \( D \) with \( N \) increases in importance as \( \gamma_v \) increases and it is essentially located in the first 10 cycles. These results are in satisfactory agreement with previously published results on Toyoura sand [Tatsuoka et al., 1978; Koruiko, 1980; Lo Presti et al., 1997].

6.6. Effect of frequency, \( f \), on damping ratio, \( D \)

The influence of frequency of cyclic loading on \( D \) is shown in Fig. 11 for \( \sigma'_{vc} = 90 \) kPa and \( \gamma_c = 0.0055\% \) (test A1 in Tab. 1). At this \( \gamma_c \) value, which is below the volumetric threshold shear strains for sands, damping ratio is practically not affected by the number of cycles. In Fig. 11 the stress-strain loops corresponding to two different frequencies, namely \( f = 0.04 \) Hz and \( f = 0.42 \) Hz are plotted. For comparison, these stress-strain loops have been plotted shifting the recorded ones in the vertical and horizontal direction such that the tips of the loops are located at the same points. It is clearly noted that both loops match perfectly. Therefore damping ratio, which is proportional to the area \( \Delta W \) enclosed by the loop, is independent of the frequency of cyclic loading in the frequency range investigated. For convenience, the average strain rates, \( \dot{\gamma} \), corresponding to the applied frequencies are also reported in the figures.

These results are in agreement with those indicated by other investigators on Toyoura sand [Lo Presti et al., 1997] or other different clean sands [Kim, 1991; Stokoe et al., 1995] which reported no effect of frequency of cyclic loading on damping ratio from torsional and resonant column tests. However, it should be said that recent experimental results by Tatsuoka et al. [2000] and Di Benedetto et al. [2002] have pointed out that the effect of loading rate and its change on the stress-strain behaviour, as well as creep and relaxation phenomena, could be not negligible even for sands.

6.7. Effect of the shape of cyclic straining on damping ratio, \( D \)

The effect of the shape of cyclic straining is shown in Fig. 12 at the vertical confining effective stress \( \sigma'_{vc} = 420 \) kPa and for \( \gamma_c = 0.011\% \) (test A3 in Tab. 1). Two different shear strain-time histories have
been taken into account, the first is trapezoidal (Fig. 12a) with a period of $T = 50\ s (f = 0.02\ Hz)$ while the second is sinusoidal (Fig. 12b) with a period of $T = 1\ s (f = 1\ Hz)$. The stress-strain loops corresponding to the two different shear strain-time histories have been compared in Fig.12c. As for the effect of the number of cycles on damping ratio, the stress-strain curves have been shifted so that the tips of the loops fall on the same points. It can be seen in Fig. 12c that the areas of the two loops are almost identical, irrespective of the large change in the shape of cyclic straining. These results are in agreement with previous published results on Toyoura sand [Toki et al., 1995] and other sands [Vucetic et al., 1998b].

It must be noted that the effect of the shape of cyclic straining on D could be rigorously examined only if the period $T$ is the same among the tests compared. It is obvious that if the shape of cyclic straining is trapezoidal and if the period $T$ is very large with respect to that of the sinusoidal shape of cyclic straining, as it is the case in Fig. 12a, the area of the loop and thus damping ratio could considerably increase due to relaxation phenomena. However, as it can be seen in Fig. 12c, these relaxation phenomena for Toyoura sand are practically negligible regardless the large change in the period $T$ between the two considered waveforms.
Fig. 10 - Effect of number of cycles, $N$, on shear modulus, $G_s$, and damping ratio, $D$, at different $\gamma_c$: (a) $\gamma_c = 0.0027-0.0032\%$, (b) $\gamma_c = 0.012-0.014\%$, and (c) $\gamma_c = 0.027-0.052\%$.

7. Comparison between the Toyoura Sand results obtained in the DSDSS device and those obtained in other laboratory investigations

In the preceding sections the DSDSS tests results on Toyoura sand expressed in terms of shear modulus and damping ratio has been closely examined to demonstrate the capability of the cyclic testing apparatus. It is of great interest now to compare the DSDSS test results with those obtained by other different devices.

As already mentioned, Toyoura sand has been tested extensively in the laboratory [Iwasaki and Tatsuoka, 1977; Iwasaki et al., 1978; Tatsuoka et al., 1978; Korkusho, 1980; Teachvorasinsuk et al., 1990; Jamioikowski et al., 1995; Pallara, 1995; Toki et al., 1995; Lo Presti et al., 1997; Ionescu, 1998; Yamashita et al., 2000] and, more recently, it has been selected as test material for an international round-robin test that involved many geotechnical laboratories disseminated in several countries [Yamashita et al., 2001]. Laboratory tests include
bender elements (BE), resonant column (RC), cyclic triaxial (CTX), monotonic loading torsional shear (MTS), and cyclic loading torsional shear (CTS) tests.

7.1. Comparison of $G_{\text{max}}$ data

The comparison between the normalized $G_{\text{max}}/F(e)$ data of Toyoura sand obtained with the DSSDSS device and those available in the literature from different laboratory tests is shown in Fig. 13. The void ratio function $F(e)$ proposed by IWASAKI and TATSUMO [1977] for sands and reported in Eq. (2) was used to normalize the data. The IWASAKI and TATSUMO [1977] relationship is also plotted in Fig. 13 as a solid line.

The $G_{\text{max}}/F(e)$ data are plotted against $\sigma'_m$ in Fig. 13. An overall agreement can be recognized even if a certain scatter between the experimental data is also apparent. The available data from literature are spread over a range whose average is represented by the IWASAKI and TATSUMO relationship. The general trend of the DSSDSS data is consistent with the available data. However, it is noted that the $G_{\text{max}}/F(e)$ data by DSSDSS tests plot in correspondence of the lower boundary of the whole set of data. One possible explanation for this difference is that the wire-reinforced membranes enclosing the specimens are not stiff enough to ensure no lateral strains during consolidation and shear. In this situation, the coefficient of lateral pressure is lower than would be expected for an at-rest coefficient $K_0$, thus resulting in lower lateral stress. Consequently, the mean stress operative during shear would be lower than that operative in $K_0$ conditions, resulting in lower stiffness. Similar observations have been reported previously in the literature from simple shear tests on sands by YOUNG and CRAVEN [1975] and by BUDHU [1985] and on undisturbed soft clay by TALESNICK and FRYDMAN [1991].

It should be pointed out that the scatter of the data can also be the result of other factors, such as different loading and boundary conditions (strain rate, mode of shear, drainage conditions, etc.), as well as different testing devices of the same type at different laboratories.

7.2. Comparison of $G/G_{\text{max}} - \gamma_c$ and $D - \gamma_c$ data

The normalized shear modulus reduction, $G/G_{\text{max}} - \gamma_c$, and damping ratio, $D - \gamma_c$, data obtained from different laboratory tests are plotted in Figs. 14 and 15, respectively.

In Fig. 14, the agreement between the $G/G_{\text{max}} - \gamma_c$ data points obtained from different studies is extremely good. The comparison has been made at comparable vertical confining effective stress of approximately 100 kPa. Such a good agreement con-
firms that the normalized shear modulus reduction curves, $G_s/G_{\text{max}} - \gamma_c$, of sands are not very sensitive to the factors that otherwise significantly affect the values of $G_{\text{max}}$.

In Fig. 15, the agreement between the $D - \gamma_c$ data obtained under similar loading conditions in different laboratories is good too, considering the inevitable uncertainties due to the differences between cyclic and resonant column tests in terms of test conditions and data reduction methods. As a matter of fact, the $D - \gamma_c$ data points from RC and CTS tests fall within the range obtained from DSDSS test results at different vertical confining stresses.

**Conclusions**

An experimental study to investigate the shear modulus and damping ratio of Toyoura sand at small to medium cyclic shear strain amplitudes was carried out. A peculiar double specimen direct simple shear (DSDSS) device was utilized for testing because of its capability to investigate soil behavior characteristics in the small as well as intermediate strain range. The effects of cyclic shear strain amplitude, $\gamma_c$, vertical effective consolidation stress, $\sigma_{\text{vc}}$, number of cycles, $N$, frequency of cyclic loading, $f$, and shape of cyclic straining on the maximum shear modulus, $G_{\text{max}}$, secant shear modulus, $G_s$, and the damping ratio, $D$, were investigated and systematically presented.

The experimental data confirmed the findings in the literature about the influence of the above mentioned parameters on the shear modulus and damping ratio of Toyoura sand.

The $G_{\text{max}}/F(e)$ values are significantly affected by the mean effective consolidation stress, $\sigma_{\text{me}}$. As expected, a linear relationship between $G_{\text{max}}/F(e)$ and $\sigma_{\text{me}}$ has been obtained in a log-log plot, with $G_{\text{max}}/F(e)$ consistently increasing as $\sigma_{\text{me}}$ increases.
However, the $G_{\text{max}}/F(c)$ data, at a given $\sigma'_{\text{ve}}$, plot in correspondence of the lower bound of the range of data from literature. It has been speculated that the flexibility of the membranes surrounding the specimens could be the major reason of the slightly lower $G_{\text{max}}$ or $G_{\text{max}}/F(c)$ values estimated with respect to those obtained using other devices.

The trends of the $G_s - \gamma_c$, $G_s/G_{\text{max}} - \gamma_c$ and $D - \gamma_c$ relationships with $\sigma'_{\text{ve}}$ were obtained as expected for sandy soils. As $\sigma'_{\text{ve}}$ increases, $G_s - \gamma_c$ relationship plots consistently higher, $G_s/G_{\text{max}} - \gamma_c$ relationship plots higher, while the $D - \gamma_c$ relationship plots lower. These trends highlight the importance of confining pressure on nonlinear shear modulus and damping relationships of sands.

The effect of the number of cycles, $N$, on $G_s$ and $D$ becomes important as the shear strain level increases. At $\gamma_c$ around 0.01%, it has been found that as $N$ increases $G_s$ moderately increases at a given confining stress; on the other hand, as $N$ increases $D$ significantly decreases at a given confining pressure, with most change occurring in the first ten cycles.

The effect of frequency of cyclic loading, $f$, on $D$ has been investigated in the range of frequencies between approximately 0.01 and 1.0 Hz, at a given vertical confining stress. It has been found that $D$ is independent on $f$, provided that the effect of number of cycles and the strain amplitude is taken into account in the comparison. Further, it has been found that the shape of cyclic straining has practically no effect on damping ratio.

The experimental program carried out enabled the comparison between the DSDSS tests results and those available on Toyoura sand from other laboratory tests, such as bender elements, resonant column, cyclic triaxial, monotonic and cyclic torsional shear tests. Despite of the inherent differences in the boundary conditions, experimental procedures and interpretation criteria, the reported laboratory values of shear modulus and damping show a very satisfactory agreement.

Acknowledgments

The research was carried out with the financial support of Ministero Università e Ricerca Scientifica e Tecnologica (M.U.R.S.T., 60%). This support is gratefully acknowledged. The authors would like to thank Prof. Lo Presti of the Politecnico di Torino for providing the Toyoura sand, Prof. Vucetic of the University of California at Los Angeles (UCLA) where the testing was conducted and Dr. Macan Do-roudian for helping in conducting the tests.

References


Armadi M. (1991) – Caratteristiche di deformabilità della sabbia del Ticino e di Toyoura da prove di taglio torsionale e colonna risonante. Tesi di Laurea, Di-
partimento di Ingegneria Strutturale, Politecnico di Torino.


KIM D.S. (1991) - Deformational characteristics of soils at small to intermediate strains from cyclic tests. Ph.D. thesis, University of Texas at Austin, USA.


Cyclic properties of Toyoura Sand at small to medium strains in simple shear test


Proprietà cicliche della sabbia di Toyoura a piccole e medie deformazioni da prove di taglio semplice

Sommatto

Sono presentati i risultati di prove di taglio semplice ciclico effettuate sulla sabbia di Toyoura a piccole e medie deformazioni. I risultati sperimentali sono espressi in termini di modulo di taglio massimo (Gmax), modulo di taglio secco (Gd), modulo di taglio normalizzato (Gd/G declarato) e fattore di smorzamento (D).

L’obiettivo dello studio è duplice: (a) analizzare in modo sistematico i risultati sperimentali con particolare attenzione all’influenza dell’ampiezza della deformazione di taglio ciclica (\(\varepsilon\)), della tensione verticale efficace di consolidazione (\(\sigma_v\)), della frequenza (f) e del numero di cicli (N) sul modulo di taglio e sul fattore di smorzamento; (b) confrontare i risultati ottenuti con quelli disponibili in letteratura, relativi a prove effettuate con altre apparecchiature di laboratorio cicliche e dinamiche.

Keywords

Sand, laboratory testing, simple shear test, cyclic loading, shear modulus, damping ratio, vertical stress, number of cycles, frequency of cyclic loading.