

# Numerical and experimental comparison between two-station and multistation methods for spectral analysis of surface waves

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## Summary

The popularity of surface wave methods in shallow geophysics and geotechnical engineering is increasing because of their cost effectiveness in testing hard to sample soils. The soil stiffness profile is obtained with an inversion process starting from the dispersive behaviour of Rayleigh waves. Typically the experimental dispersion curve is evaluated using a two-station procedure based on measuring the phase difference between impulsive signals recorded at two receivers (SASW test). In this paper a comparison between the results obtained with such approach and those relative to a multistation procedure based on frequency-wavenumber analysis is presented. The multistation approach introduces important improvements, reducing the testing time and the noise effects.

Both synthetic signals and experimental data have been used for a large spectrum comparison, whose main objective is to establish the differences in the experimental dispersion curves obtained with the two approaches.

Finally the stiffness profiles evaluated from the inversion of the multistation dispersion curve are compared to the results of seismic borehole methods.

## 1. Introduction

Many relevant problems in Earthquake Geotechnical Engineering and Soil Dynamics, including site response analysis and design of vibrating machine foundations, need a sufficiently accurate evaluation of the soil stiffness properties. In particular the stiffness at very low strains is a fundamental parameter for every dynamical soil model and its evaluation with laboratory test can be problematic. This aspect is particularly important in hard-to-sample materials such as cohesionless soils, in which obtaining undisturbed samples is difficult and very expensive.

In situ small strain stiffness is often obtained with seismic methods, which are based on wave propagation. Boreholes seismic methods such as the Cross-Hole and the Down-Hole tests are very accurate but costly and time consuming. Non-invasive surface wave based methods offer an acceptable accurateness without the need for boreholes.

The stiffness profile is estimated with an inversion process using the dispersive properties of surface waves. The main advantage is that Rayleigh and Love waves can be detected on the ground surface. Early applications were developed by seismologists for the characterisation of the Earth's crust [DORMAN and EWING, 1962; AKI and RICHARDS, 1980].

On the engineering side, small scale simplified applications were proposed for soil and pavement

characterisation using the Steady State Rayleigh method [JONES, 1958].

But the wide spread of surface wave based methods in geotechnical engineering started from the middle Eighties, when the researchers of the University of Texas at Austin developed the SASW (Spectral Analysis of Surface Waves) method. This method combined a rigorous approach for the inversion process with a strong testing time reduction in the field [NAZARIAN and STOKOE, 1984]. Successively several similar techniques have been proposed for soil characterisation, either based on the use of active sources or micro-tremors [see FOTI, 2000].

In its original configuration the SASW method uses an impact source and a couple of receivers connected to a signal analyser [NAZARIAN and STOKOE, 1984]. Such simple test setup has given a strong impulse for the diffusion of the method, but it has some inherent drawbacks, which are related to the necessity of repeating the test in several different configurations and to some difficulties in data interpretation, as it will be explained in Section 2.1.

The use of multistation testing configurations overcomes most of these limitations. Multistation detection and analysis of impulsive signals have been successfully applied for soil characterisation on large-scale projects [GABRIELS *et al.*, 1987; McMECHAN and YEDLIN, 1981], but they are not commonly used for geotechnical applications.

This paper is focused on the comparison between the multistation approach and the traditional

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two-station approach in determining the dispersion curve with the SASW method. Both synthetic and experimental data are used to have a comprehensive picture of the differences between the two methods for geotechnical site characterisation.

The paper is composed of three main sections: in the first one both the two-station and the multi-station methods are presented and briefly discussed. The second section contains the results of the numerical simulations, while the last one presents some case histories in which experimental data are interpreted using both techniques. Finally the experimental dispersion curves obtained with the multistation method have been inverted to yield the stiffness profiles and the results have been compared with DHT and CHT data.

## 2. The use of Rayleigh waves for soil characterization

Rayleigh waves propagate along a free boundary of a medium and induce motion in a limited skin-depth that is a function of the wavelength. In a homogenous elastic halfspace, they propagate with a constant velocity, which is function only of the elastic parameters. In heterogeneous elastic media their velocity of propagation is a function of frequency and such dependence is strictly related to the stiffness variation with depth. Hence it is possible with a process of inversion to extract information about the stiffness profile from the relationship between velocity of propagation of Rayleigh waves and frequency.

Such inversion process requires the choice of a reference model for the medium and the obtained results will be strongly dependent on such a choice. A horizontally layered elastic model is usually adopted for soil deposits. The results of a SASW inversion analysis can be considered reliable only if there is a sufficient correspondence of the model to the actual geometry of the soil deposit.

Some techniques based on the acquisition and analysis of microtremors have been proposed [TOKIMATSU, 1995], but typical engineering applications of surface wave methods involve the generation of a wavefield using a source acting on the ground surface. The source can be either impulsive, as for example a sledge-hammer or a weight-drop system, or controlled, able to reproduce a harmonic input in the ground. The advantage of the latter option is the possibility of increasing the signal to noise ratio [RIX, 1988], but it requires more sophisticated and expensive equipment.

The wavefield generated by a source acting on the ground surface is not uniquely composed of Rayleigh waves, but also of body waves, which represent a disturbance. Nevertheless Rayleigh waves typ-

ically dominate the wavefield because of the following physical aspects:

- a great part of the energy transmitted to the ground by a point source goes into Rayleigh waves. For instance for harmonically vibrating circular footings over an elastic homogeneous halfspace at low frequencies of operation, about 2/3 of the energy goes into Rayleigh waves and the remaining portion is subdivided between compression and shear waves [RICHART *et al.*, 1970];
- the energy attenuation associated to geometrical spreading is lower for Rayleigh waves than for body waves. For a point source, body waves in an infinite elastic medium attenuate as  $1/r$ , while along a free surface compression and shear waves attenuate as  $1/r^2$  [EWING *et al.*, 1957]. Rayleigh waves spread along a cylindrical wavefront and hence their attenuation is proportional to  $1/\sqrt{r}$ .

For the aforementioned reasons the effects of body waves are generally sensible only in the nearby of the source, hence they are usually named near-field effects.

The particle motion generated by the source is detected at one or more points on the ground surface and analysed to extract the experimental dispersion curve, i.e. the relationship between velocity of propagation of surface waves and frequency. A variety of techniques have been proposed for this purpose [see FORT, 2000]. In the following the two-station procedure of the SASW test and the multistation procedure based on the frequency-wavenumber analysis are discussed.

### 2.1. The SASW method

The typical configuration for the SASW test is represented in Fig. 1. An impact source creates a wave-train, which has components in a broad frequency range. The ground motion is detected by a pair of receivers, which are placed along a straight line passing from the source, and the signals are then analysed in the frequency domain. The phase velocity  $V_R$  is obtained from the phase difference of the signals using the following relationship:

$$V_R(\omega) = \frac{\omega}{\Theta_{12}(\omega)} \cdot X \quad (1)$$

in which  $\Theta_{12}(\omega)$  is the cross-power spectrum phase,  $\omega$  is the angular frequency and  $X$  is the inter-receiver spacing (Fig. 1).

One critical aspect of the above procedure is the influence of signal-to-noise ratio. Indeed the measurement of phase difference is a very delicate task. The necessary check on the signal to noise ratio is usually accomplished using the coherence function [SANTAMARINA and FRATTA, 1998], whose value is

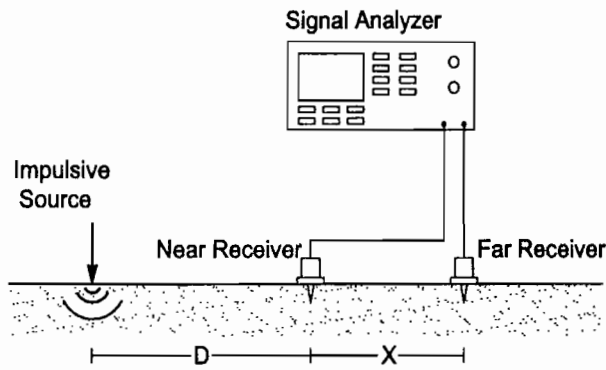


Fig. 1 – Two-receiver configuration (SASW test).

Fig. 1 – Configurazione di prova a due ricevitori (metodo SASW).

equal to 1 for linearly correlated signals in absence of noise. Only the frequency ranges having a high value of the coherence function are used for the construction of the experimental dispersion curve. It must be remarked that the coherence function must be evaluated using several pairs of signals, leading to the necessity of repeating the test using the same receiver setup.

Other important concerns are near-field effects and spatial aliasing in the recorded signals. In this respect, usually a filtering criterion (function of the testing setup) is applied to the dispersion data [GANJI *et al.*, 1998]. E.g. only frequencies for which the following relationship is satisfied are retained:

$$\frac{X}{3} < \lambda_R(\omega) < 2D \quad (2)$$

in which  $\lambda_R(\omega) = V_R(\omega)/f$  is the estimated wavelength,  $D$  is the source-first geophone distance, and  $X$  is the inter-receiver spacing (Fig. 1). Typically, the receiver positions are such that  $X$  and  $D$  are equal, in accordance to the results of some parametric studies about the optimal test configuration [SANCHEZ-SALINERO, 1987].

The above filtering criterion assumes that near fields effects are negligible if the first receiver is placed at least half a wavelength away from the source, for a given frequency in the spectral analysis. Such assumption is acceptable in a normally dispersive site, i.e. a site having stiffness increasing with depth, but it can be optimistic for more complex situations [TOKIMATSU, 1995]. For this reason and in order to avoid great loss of data, inversion methods that take into account near field effects have been proposed [ROESSET *et al.*; 1991, GANJI *et al.*, 1998].

For the aforementioned considerations a single testing configuration gives information only for a particular frequency range, which is dependent on receiver positions. The test is then repeated using a variety of geometrical configurations which include

adapting the source type to the actual configuration, i.e. lighter sources (hammers) are used for high frequencies (small receiver spacing) and heavier ones (weight-drop systems) for low frequencies (large receiver spacing). Usually five or six setups are used, moving source and receivers according to a common-receiver-midpoint scheme [NAZARIAN and STOKOE, 1984].

Typically the test is repeated for each testing configuration in a forward and reverse direction, moving the source from one side to the other with respect to the receivers. Such procedure is quite time consuming, but it is required to avoid the drift that can be caused by instrument phase shifts between the receivers, since the analysis process is based on a delicate phase difference measurement. It is also claimed that such expedient can compensate for dipping layers [NAZARIAN and STOKOE, 1984], but the model for the inversion process is quite different from the actual geometry and the relative implications on the results are not clear. Some numerical analysis on F.E.M. models have shown the shift in the dispersion curve caused by dipping layers [GUKUNSKI *et al.*, 1996].

Finally, the information collected in several testing configurations is assembled and averaged to estimate the experimental dispersion curve at the site, which will be used for the subsequent inversion process.

A very ticklish task in the interpretation of the SASW test is related to the unwrapping of the Cross-Power Spectrum phase. Indeed it is obtained in a modulo- $2\pi$ , which is very difficult to interpret and unsuitable for further processing [POGGIAGLIOLMI *et al.*, 1982]. The passage to an unwrapped (full-phase) curve is necessary for the computation of time delay as a function of frequency (see Eq. 1).

Usually some automated algorithms are applied for this task [POGGIAGLIOLMI *et al.*, 1982], but external noise can produce fictitious jumps in the wrapped phase, which drastically damage the results. Not always the operator can correct such unwrapping errors on the basis of judgement and in any case it is a subjective procedure, which precludes the automation of the process. An automated procedure based on a least-square interpolation of the cross-power spectrum phase has also been proposed [NAZARIAN and DESAI, 1993].

## 2.2. Multistation approaches

The use of a multistation testing configuration has some evident advantages because of the greater amount of simultaneously collected information.

Such information can be profitably used for a rapid and stable estimation of the dispersion curve, as will be shown in the following. Moreover using a

multistation scheme and a controlled sweep-sine source, RIX *et al.* [2000; 2001] have shown that it is also possible to infer from surface waves measurements an estimate of the small strain damping ratio as a function of depth.

The testing setup that will be considered in the following consists of an impulsive source and an array of equally spaced geophones (Fig. 2).

From the field data in the time-space domain, the dispersion curve for Rayleigh waves can be easily obtained using a double transform and a picking algorithm, either in the frequency-wavenumber domain or in the frequency-slowness domain. A brief explanation of the procedure for the frequency-wavenumber analysis follows.

For a point source acting on the ground surface in a horizontally layered medium, the particle motion associated to Rayleigh wave propagation can be written as the superposition of modal contributions [AKI and RICHARDS, 1980]:

$$s(x, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \sum_m S_m(\omega, x) \cdot e^{i(\omega t - k_m(\omega)x)} d\omega \quad (3)$$

where  $m$  is the mode number and the factor

$$S_m(\omega, x) = I(\omega) \cdot P_m(\omega) \cdot R_m(\omega) \cdot \frac{e^{-\alpha_m(\omega)x}}{\sqrt{x}} \quad (4)$$

is a combination of instrument response  $I(\omega)$ , source spectrum  $P_m(\omega)$  and path response  $R_m(\omega)$  with geometric (represented by the factor  $\frac{1}{\sqrt{x}}$ ) and material (coefficient  $\alpha_m$ ) attenuation. Notice that all the above factor are frequency dependent.

The modal wavenumber  $k_m$  is inversely proportional to the modal phase velocity  $V_{Rm}$ :

$$k_m(\omega) = \frac{\omega}{V_{Rm}(\omega)} \quad (5)$$

The dependence of  $S_m$  on distance from the source is only related to the attenuation phenomenon. The influence of the geometrical attenuation

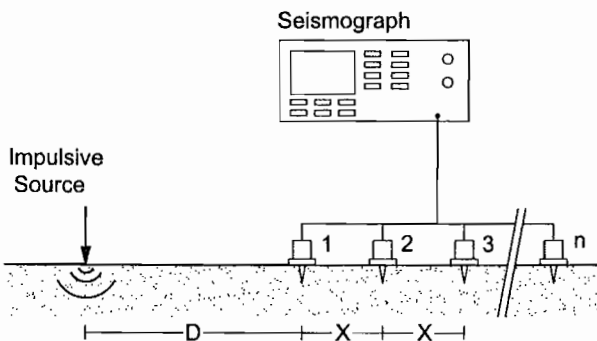


Fig. 2 – Multistation configuration ( $fk$  analysis).

Fig. 2 – Configurazione di prova multicanale (analisi  $fk$ ).

can be easily removed by multiplying each contribution by the square root of the source-receiver distance and it will be therefore neglected in the following, assuming that such correction is applied on the original data. As far as the material attenuation is concerned, its contribution can be taken out from the expression of  $S_m$ , so that the latter becomes distance independent.

Hence, applying to (3) a discrete slant stack transform [YILMAZ, 1987] and successively a discrete Fourier transform, the  $fk$  displacement spectrum can be written as [TSELENTIS and DELIS, 1998]:

$$F(\omega, k) = \sum_m S_m(\omega) \cdot \left[ \sum_{n=1}^N e^{-\alpha_m(\omega) x_n} \cdot e^{i(k - k_m(\omega)) x_n} \right] \quad (6)$$

Neglecting the material attenuation contribution, differentiating the quantity in the square bracket with respect to  $k$  and setting the results equal to zero, it comes out that the peaks in the displacement spectrum are found for  $k = k_m(\omega)$ . Furthermore it can be shown that also if the above differentiation is conducted including the material attenuation factor the conclusion is the same, i.e. the accuracy is not conditioned by material attenuation [TSELENTIS and DELIS, 1998].

Once the modal wavenumbers have been estimated for each frequency, they can be substituted in Equation 5 to evaluate the modal dispersion curves.

The proof has been outlined considering the displacement spectrum, however in the usual practice velocity transducers (geophones) are used, hence the results are typically reported in terms of velocity spectra.

Application of the  $fk$  analysis procedure is shown in Fig. 3 using synthetic data. A three-dimensional plot of the velocity spectrum is reported in Fig. 3-a. The contour plot (Fig. 3b) is a more used graphical representation because it shows the location in the  $fk$  plane of the maxima. The phase velocity of surface waves for a given frequency is evaluated from the location of the maximum in the corresponding “slice” of the spectrum (Fig. 3c). E.g. for the synthetic data shown in Fig. 3 it is possible to obtain.

$$V_{Rm}(50\text{Hz}) = \frac{2\pi \cdot f}{k|_{F=\max}} = \frac{2\pi \cdot 50}{0.97} = 324 \text{ m/s}$$

The process is repeated over the frequency range of interest. In practice the actual range of frequencies is related to the signal-to-noise ratio. Moreover the upper bound is limited by spatial aliasing, that can be easily recognised by the wrap-around of the spectrum [YILMAZ, 1987].

Similarly it can be shown [MCMEECHAN and YEDLIN, 1981] that the dispersion curve can be obtained from the spectral maxima in the frequency-slowness

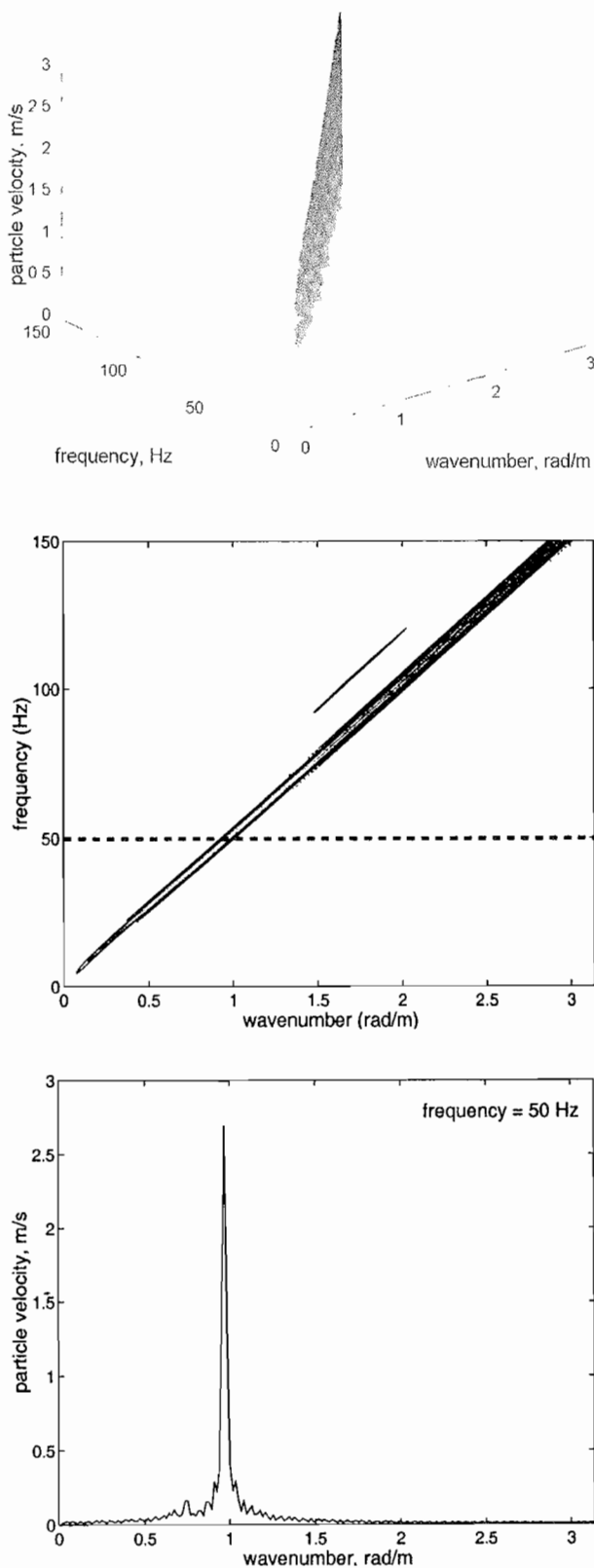


Fig. 3 – Example of  $fk$  analysis of surface waves on synthetic data: a)  $fk$  spectrum (3D plot) b)  $fk$  spectrum (contour plot) c)  $fk$  spectrum at frequency=50Hz.  
 Fig. 3 – Esempio di analisi  $fk$  delle onde superficiali su dati sintetici: a) spettro  $fk$  (rappresentazione 3D) b) spettro  $fk$  (curve di livello) c) spettro  $fk$  per frequenza=50Hz.

domain. The equivalence of the two procedures is a consequence of the Fourier Slice Theorem [SANTAMARINA and FRATTA, 1998].

It is important to point out that the above multistation procedures are directly founded on the dispersive nature of surface waves, hence they are less affected by near-field effects than the two-station analysis. Indeed the energy related to different events (reflected waves, back-scattered waves, ground roll, etc.) is located in different portions of the two dimensional space of the transformed variables [DOYLE, 1995]. This aspect has been confirmed by some comparative numerical simulations [FOTI, 2000].

The length of the testing array strongly affects the results that can be obtained using  $fk$  analysis of surface waves. If a long receiver array is used, modal dispersion curves can be easily obtained experimentally and used for the inversion process [GABRIELS *et al.*, 1987], but for shorter receiver arrays it is necessary to deal with mode superposition effects [FOTI *et al.*, 2000].

### 3. Numerical simulations

A comparative numerical analysis has been performed using Rayleigh waves synthetic seismograms, which have been generated using a computer code by R.B. Herrmann and his co-workers of S.Louis University [HERRMANN, 1996]. The impact source has been modelled as an impulsive vertical point source and the corresponding seismograms have been evaluated at a given number of detection points on the ground surface along a straight line passing through the source.

Three profiles have been analysed: Case A represents a normally dispersive medium, with soil stiffness increasing with depth without strong impedance jumps; in Case B a stiff top layer is placed above a normally dispersive medium, generating an inversely dispersive system; finally Case 3 is designed to simulate the presence of a stiff bedrock below a homogenous soil. The above profiles represent typical subsoil conditions that can be found in geotechnical engineering problems.

For the two-station SASW method, 5 receiver configurations have been used considering respectively the following values of the inter-receiver spacing: 1 m, 2 m, 5 m, 10 m and 20 m. The phase velocity has been evaluated using Equation 1 and the information has been filtered according to the criterion of Equation 2.

The multistation  $fk$  analysis has been applied on a set of 24 synthetic traces with inter-receiver spacing equal to 1m. The use of 24 traces is related to the usual number of channels of common commercial seismographs.

### 3.1. Case A

The geometrical and mechanical parameters are reported in Tab. I and the shear velocity profile is represented in Fig. 4. This two-layer over halfspace system represents a normally dispersive medium with respect to Rayleigh wave propagation (recalling that a wave is defined normally dispersive if its phase velocity is decreasing monotonically for increasing frequency [SHERIFF and GELDART, 1995]).

In such normally dispersive media, in which the stiffness increases with depth without strong imped-

Tab. I - Case A: geometry and mechanical properties.

Tab. I - Caso A: geometria e proprietà meccaniche degli strati.

Thickness (m)	$V_S$ (m/s)	$V_P$ (m/s)	Density ( $\text{kg/m}^3$ )
5	350	600	1800
10	400	700	1800
$\infty$	450	800	1800

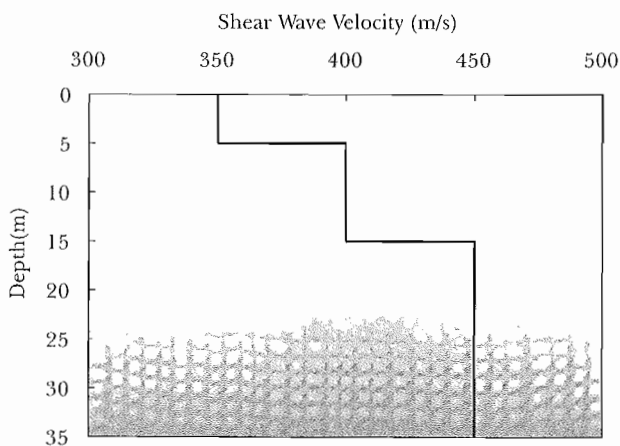


Fig. 4 - Case A: shear wave velocity profile.

Fig. 4 - Caso A: profilo di velocità delle onde di taglio.

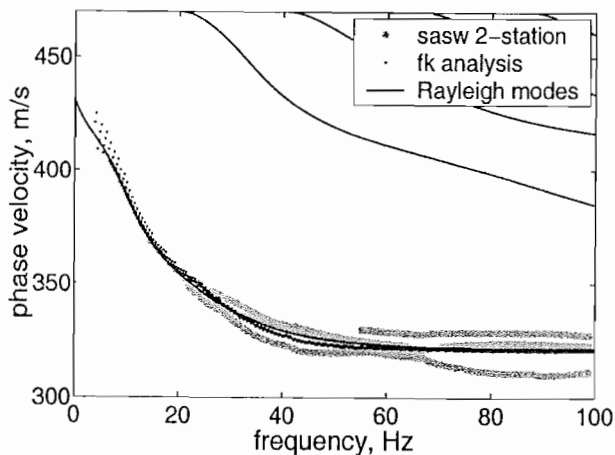


Fig. 5 - Case A: estimated dispersion curve.

Fig. 5 - Caso A: stima della curva di dispersione.

ance jumps, the fundamental mode generally dominates surface wave propagation and the higher mode effects can be neglected when interpreting SASW tests [GUKUNSKI and WOODS, 1992; Tokimatsu, 1995]. Indeed the dispersion data obtained analysing each pair of synthetic seismograms, as described above, are distributed in a narrow band close to the fundamental mode dispersion curve (Fig. 5). In this case the fk analysis gives directly an estimate of the fundamental mode, which corresponds to an average of the two-receiver data (Fig. 5). Hence for both methods the inversion process can be performed using a fundamental mode analysis.

### 3.2. Case B

To explore the effects of a stiff surface layer, the same stiffness profile of case A has been used adding a top layer as stiff as the halfspace. The same procedure of generating and analysing the synthetic seismograms has been used also for this case. The profile parameters are reported in Table II and the shear wave velocity profile is shown in Fig. 6.

Several researchers [GUKUNSKI and WOODS, 1992; TOKIMATSU, 1995] have already shown the strong influence of higher modes for inversely dispersive media. It is important to observe (Fig. 7) that in this case higher mode influence is sensible as frequency increases. Indeed the asymptotic value of the phase velocity for increasing frequency is strongly related to the first layer stiffness. Such asymptotic behaviour can be obtained only with a continuous switch of dominating mode from the fundamental one towards higher ones.

Both the two-station method and the multistation fk analysis yield a dispersion curve, which is determined by mode superposition (Fig. 7). Following the suggestion of TOKIMATSU [1995], we will refer to such phase velocity as apparent velocity of Rayleigh waves. It can be shown that, over long distances, the difference of group velocity between modes produces a mode separation effect and, using the fk analysis, it is possible to obtain the dispersion curves of several distinct modes [GABRIELS *et al.*, 1987; FOTI *et al.*, 2000].

As for case A, also in this case the multistation fk analysis produces a unique estimate in comparison to the sparse values that are obtained using the two station procedure over several receiver pairs.

The implications of the above results on the inversion process must be carefully evaluated. It is important to point out that, independently on the procedure used to estimate the experimental dispersion curve, a fundamental mode approach can not be used. Indeed for an inversely dispersive medium, the higher modes play a strong role in the propagation of surface waves and their influence can not be neglected. Hence for an accurate evaluation of soil

Tab. II – Case B: geometry and mechanical properties.  
 Tab. II – Caso B: geometria e proprietà meccaniche degli strati.

Thickness (m)	$V_S$ (m/s)	$V_P$ (m/s)	Density ( $\text{kg/m}^3$ )
3	450	800	1800
5	350	600	1800
10	400	700	1800
$\infty$	450	800	1800

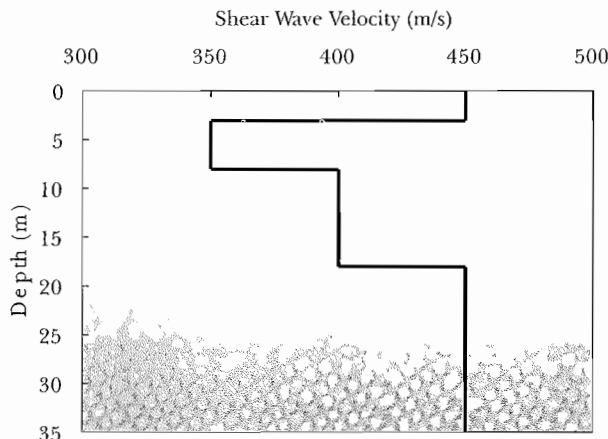


Fig. 6 – Case B: shear wave velocity profile.  
 Fig. 6 – Caso B: profilo di velocità delle onde di taglio.

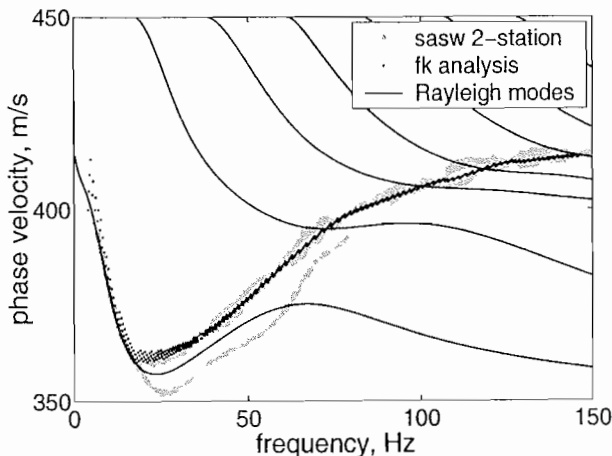


Fig. 7 – Case B: estimated dispersion curve.  
 Fig. 7 – Caso B: stima della curva di dispersione.

parameters, an inversion process based on a consistent definition of the apparent phase velocity must be used [LAI, 1998].

### 3.3. Case C

The last synthetic profile represents the typical case of a soil layer over a bedrock (see Tab. III and Fig. 8). In this case the stiffness is increasing with depth and hence the profile is normally dispersive. Nevertheless the strong impedance jump makes

higher modes significantly influence on the apparent phase velocity. In particular the second mode is important in the low frequency range (Fig. 9), in opposition to the situation of case B in which higher modes were influent in the high frequency range (Fig. 7). Hence also in this case it is necessary to consider mode superposition in the interpretation. The comparison between the two methods of analysis lead to the same conclusions of the previous cases.

## 4. Experimental results

Some case histories are presented in the following to validate the conclusions obtained with the synthetic data. The case histories have been selected

Tab. III – Case C: geometry and mechanical properties.  
 Tab. III – Caso C: geometria e proprietà meccaniche degli strati.

Thickness (m)	$V_S$ (m/s)	$V_P$ (m/s)	Density ( $\text{kg/m}^3$ )
10	300	500	1800
$\infty$	900	1550	1800

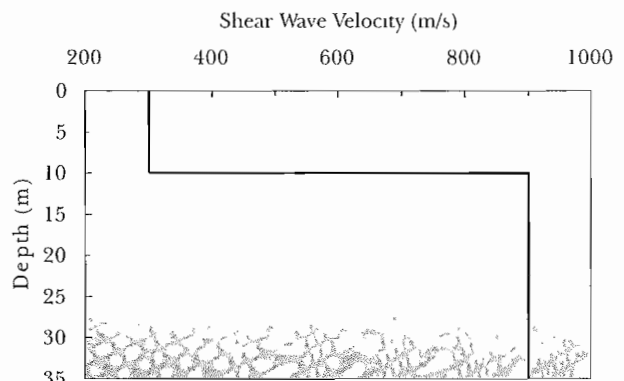


Fig. 8 – Case C: shear wave velocity profile.  
 Fig. 8 – Caso C: profilo di velocità delle onde di taglio.

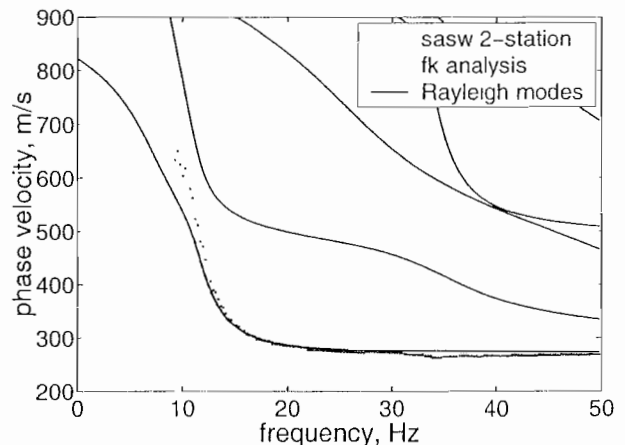


Fig. 9 – Case C: estimated dispersion curve.  
 Fig. 9 – Caso C: stima della curva di dispersione.

such that the real subsoil conditions are similar to the fictitious profiles used for the numerical simulations.

Data have been collected using a 24 channels seismograph Mark6 (by ABEM Ltd) and 24 vertical geophones (4.5 Hz natural frequency). The main purpose of this paper is to compare the experimental dispersion curves that are obtained using two-station and multistation methods. Nevertheless for the sake of completeness also the stiffness profiles, which are obtained from the inversion of the experimental dispersion data, are presented and compared to the results of borehole tests.

The dispersion curves of the multistation  $f/k$  analysis have been used for the inversion process, assuming a horizontally layered linear elastic model for the soil. Such a model is characterised by 4 parameters for each layer (e.g. thickness, density, shear modulus and Poisson ratio), except for the lowermost halfspace, which has no fixed thickness. As usual in soil characterization using surface waves data, density and Poisson ratio of each layer have been assumed a-priori, on the basis of general information on the specific site. Such procedure is generally accepted because it has been shown that such system parameters have a minor influence on the dispersion curve [NAZARIAN, 1984].

Obviously, in the comparison of surface waves and borehole tests results, the inherent differences in volume of tested soil must be kept in mind. Indeed the cross-hole measurement is made between very close points, the surface wave test measures the average properties over a section of several tens of meters and the down-hole method measures average soil properties along depth. This aspect obviously affects the resolution that can be expected from each one of the above methods. Clearly the surface wave method has a lower resolution, e.g. it is not able to detect thin layers at great depth. Nevertheless it has several advantages related to time, cost and possibility of testing the soil without any drilling and casing disturbance.

#### 4.1. Site A

The test site is located in Saluggia (VC) in the northern part of Italy, close to the Dora Baltea River and it is part of a large flat area of fluvial sediments. The soil is composed basically of gravels and gravelly sands, with the presence of fine sand and clayey silt, in the form of lenses. The water table is at very shallow depth, between 2 and 3 meters below the ground surface. The results of a CH test at the site are available from a previous geotechnical survey.

The data have been collected using two different test arrangements having respectively receiver spacing equal to 1 m and 3 m and with the source-

first geophone spacing equal to the inter-receiver spacing. For both testing configurations, impact sources have been used, respectively a 6kg sledgehammer and a 130kg weight-drop system (height 3m above the ground level). The two configurations have been chosen to investigate respectively the high and the low frequency ranges. An example of estimated  $f/k$  spectrum is reported in Fig. 10. For the two-station procedure the signal pairs at the following inter-receiver distance have been used: 1 m, 2 m, 5 m, 10 m and 20 m.

Experimental dispersion curves obtained with the two-station and with the multistation approach are reported in Fig. 11. The conclusions of the numerical simulations are substantially confirmed by the experimental data: the multistation dispersion curves represent the central values of the results obtained with the two-station approach.

The dispersion curve obtained by the  $f/k$  analysis of the two shot gathers (Fig. 11) shows that in the frequency range in which there is an overlap of information the agreement is very good, confirming that the estimate is stable. Globally the frequency range between 8 and 68 Hz is explored. For low frequencies (below 15Hz) only information from the 3 m gather are available, while for high frequencies (above 35Hz) the other gather supplies the information.

The experimental dispersion curve implicitly contains the information relative to the geometry and the mechanical parameters of the soil deposit. An inversion process based on the fundamental mode has been used in this case, because the site is clearly normally dispersive and only the fundamental mode has been recovered from the analysis of the seismic gathers. The starting profile for the inversion process has been selected on the basis of approximate procedures for the estimate of the shear

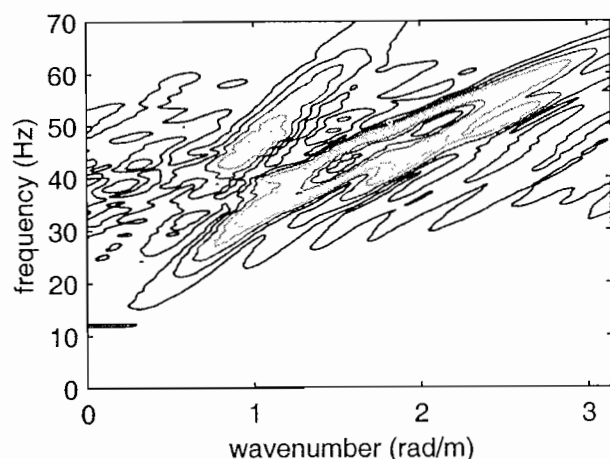


Fig. 10 – Site A:  $f/k$  spectrum (receiver spacing: 1m; source: sledge-hammer).

Fig. 10 – Sito A: spettro  $f/k$  (spaziatura ricevitori: 1m; sorgente: mazza 6kg).



wave velocity as a function of depth [GANJI *et al.*, 1998; FOTI, 2000].

The numerical dispersion curve, corresponding to the final iteration of the inversion procedure, shows a good agreement with the experimental one (Fig. 12).

The resulting shear wave velocity profile is compared to the results of the cross-hole test in Fig. 13. Clearly the surface wave based test pay the lack of resolution if compared to the borehole measurement, nevertheless it is important to note that the average stiffness has been successfully estimated.

4.2. Site B

In the second testing site a stiff top layer (a bituminous road pavement) is present over a natural soil. Unfortunately no detailed information from other tests are available for this site and hence a comparison is not possible. A later excavation has shown the presence of bedrock at a quite shallow depth.

Since in this case the depth of interest for the survey was less than in the previous case, a testing configuration having a short receiver spacing (0.5 m) has been selected. The source used for this survey was the 6 kg hammer, which was stroke at 1 m and at 5 m from the first receiver. For the two-station procedure the signal pairs at the following inter-receiver distance have been used: 1 m, 2 m, 4 m, 6 m and 8 m.

The presence of the stiff top layer makes this profile inversely dispersive and the obtained experimental dispersion curve (Fig. 14) must be considered as the superposition of several modes of propagation.

In this case a different approach must be used for the inversion process. A numerical estimate of the apparent velocity that is associated to mode superposition can be derived from the numerical analysis of the wavefield generated by a point source at short distances from the source itself. An iterative inversion procedure based on such estimate has been used to obtain the shear wave velocity profile (Fig. 16). The numerical apparent phase velocity corresponding to the profile is compared with the experimental one in Fig. 15.

4.3. Site C

Site C is located in the Tuscany region, in the central part of Italy. The geology of the area is such that the presence of a quite stiff soil was expected at shallow depth below a softer layer (about 10m) [FERRINI, 2000]. A DH survey has been performed after the surface wave test.

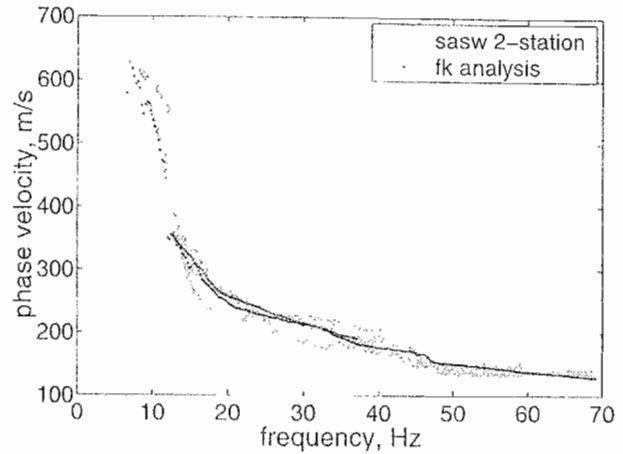


Fig. 11 – Site A: experimental dispersion curve.  
Fig. 11 – Sito A: curva di dispersione sperimentale.

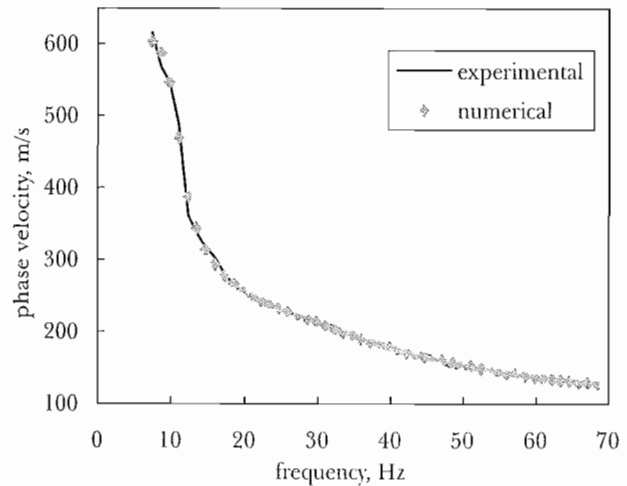


Fig. 12 – Site A: experimental vs. numerical dispersion curve.  
Fig. 12 – Sito A: confronto tra curve di dispersione sperimentale e numerica.

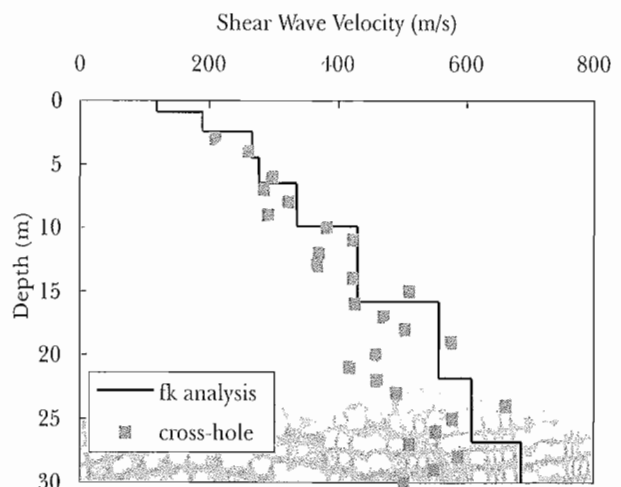


Fig. 13 – Site A: shear wave velocity profile.  
Fig. 13 – Sito A: profilo di velocità delle onde di taglio.

Two test setups have been used with inter-receiver spacing respectively of 1 m and 2 m. For the two-station analysis the signal pairs at the following inter-receiver distance have been selected: 2 m, 5 m, 10 m, 12 m, 16 m and 24 m.

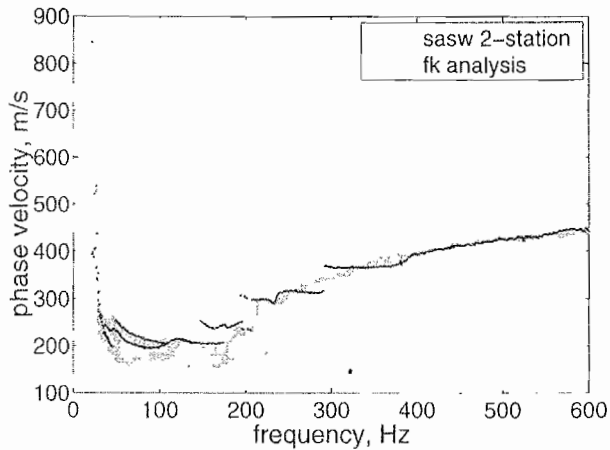


Fig. 14 - Site B: experimental dispersion curve.

Fig. 14 - Sito B: curva di dispersione sperimentale.

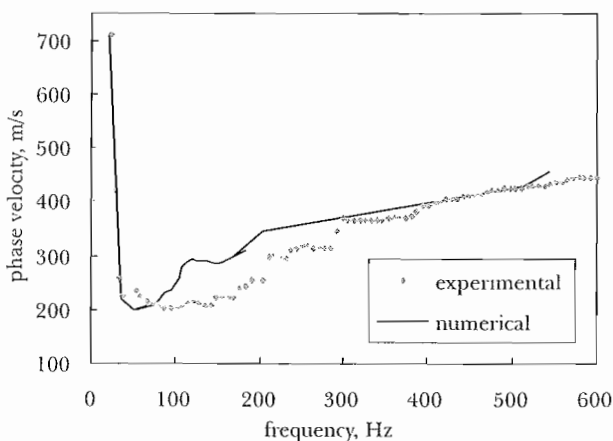


Fig. 15 - Site B: experimental vs. numerical dispersion curve.

Fig. 15 - Sito B: confronto tra curve di dispersione sperimentale e numerica.

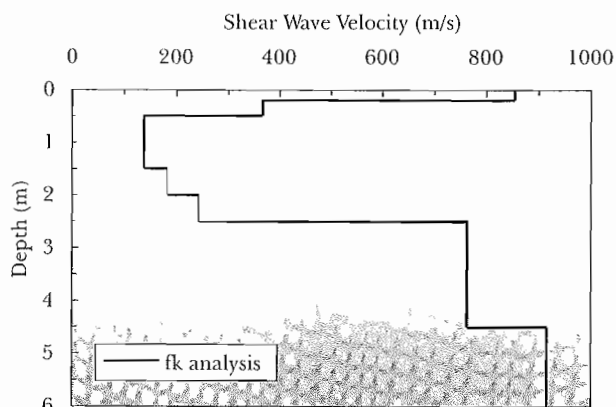


Fig. 16 - Site B: shear wave velocity profile.

Fig. 16 - Sito B: profilo di velocità delle onde di taglio.

The experimental dispersion curves obtained with the two-station and the multistation methods are reported in Fig. 17. For a case in which a soil over a bedrock is considered, mode superposition causes an increment of the apparent phase velocity with respect to the fundamental mode value in the low frequency range (see Fig. 9). Thus also in this case a consistent inversion method requires higher modes to be considered, otherwise the estimated stiffness of the lower layers would be strongly over-estimated. The results of the inversion process are reported in Fig. 18 and Fig. 19. This example shows the necessity of considering mode superposition also in normally dispersive sites, if abrupt changes in stiffness are present.

The obtained stiffness profile compares well with DHT results below 3 m (note moreover that the results of the DHT cannot be considered accurate in the first meters because of induced disturbance in boring and casing the borehole).

## 5. Conclusions

Surface waves based methods are increasing their popularity, which is due mainly to the possibility of testing undisturbed soils avoiding the costs related to boreholes. In this paper a wide spectrum comparison between two-station and multistation methods has been presented, considering both synthetic and experimental data. The obtained dispersion curves can be considered practically equivalent for the purposes of the inversion process aimed at estimating the stiffness profile.

Nevertheless the  $fk$  analysis implies a reduced number of testing and interpretation steps with respect to the two-station method, resulting in a considerable saving of time. The testing time in situ is strongly reduced because only one or two testing configurations are required. Moreover it must be considered that the  $fk$  analysis is less influenced by external noise, which often prevent two-station data to be correctly interpreted, and, as a consequence, there is much less necessity of stacking experimental signals. For instance it must be considered that the experimental data presented in this paper have been obtained with a single shot for the  $fk$  analysis, while the traditional two-station interpretation requires a minimum of 3-5 stacks for each testing configuration.

The data processing is much faster and it requires only a very limited operator judgement resulting in a strong automation of the process. This is also due to the fact that  $fk$  analysis yields directly a single dispersion curve, with no need for averaging.

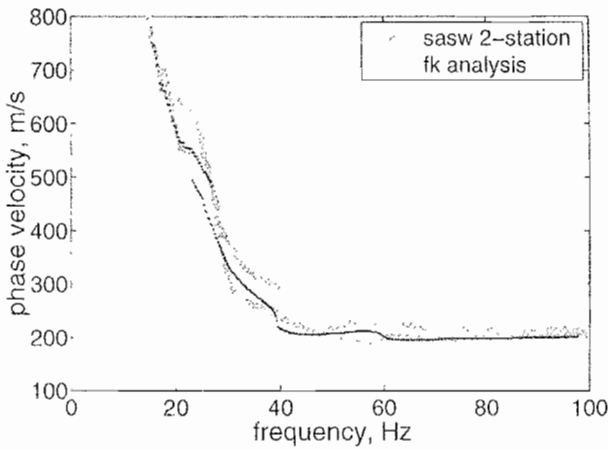


Fig. 17 – Site C: experimental dispersion curve.  
Fig. 17 – Sito C: curva di dispersione sperimentale.

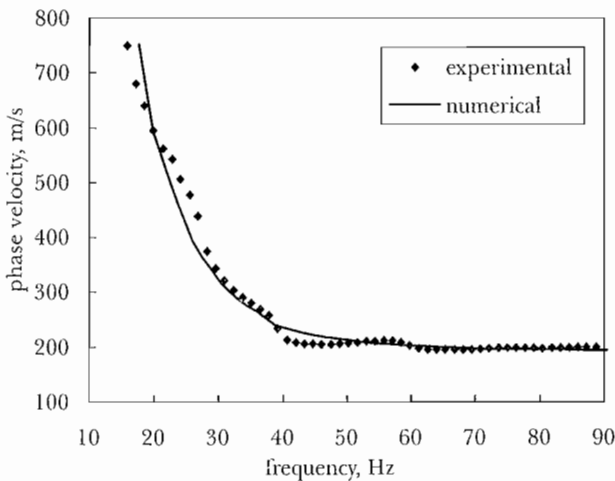


Fig. 18 – Site C: experimental vs. numerical dispersion curve.  
Fig. 18 – Sito C: confronto tra curve di dispersione sperimentale e numerica.

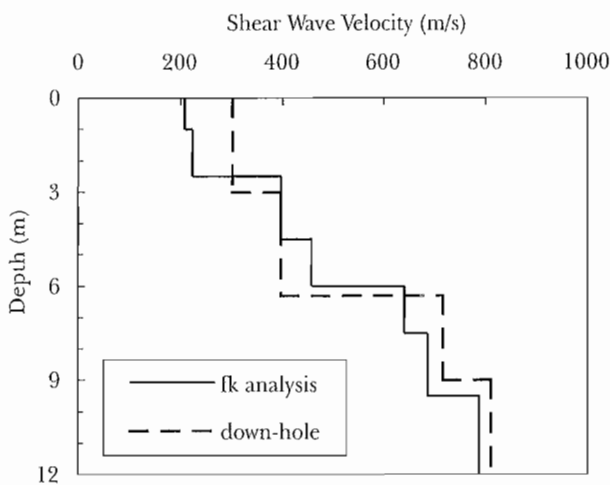


Fig. 19 – Site C: shear wave velocity profile.  
Fig. 19 – Sito C: profilo di velocità delle onde di taglio.

Finally, the stiffness profiles obtained from the inversion of surface wave data show a satisfactory agreement with bore-hole test results.

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## Confronto numerico e sperimentale tra il metodo a due stazioni e il metodo multi-stazione nell'analisi spettrale delle onde superficiali

### Sommario

I metodi di indagine geofisica basati sulla propagazione delle onde di Rayleigh consentono una ricostruzione sufficientemente dettagliata delle variazioni di rigidità con la profondità. Essi si stanno recentemente diffondendo nell'ambito della geofisica applicata e dell'ingegneria geotecnica, per effetto dei vantaggi economici ed ingegneristici derivanti dalla possibilità di evitare fori di sondaggio.

Nel metodo SASW, che rappresenta la variante più diffusa in ambito geotecnico di tali metodi, i dati sperimentali vengono collezionati in sito con una configurazione di prova a due ricevitori ed analizzati con una procedura basata sulla differenza di fase tra i segnali.

Nel presente articolo viene presentato un confronto ad ampio spettro tra tale metodo e una metodologia di prova a più ricevitori basata sull'analisi nel dominio frequenza-numero d'onda. Quest'ultima consente una notevole riduzione dei tempi di acquisizione ed interpretazione e presenta il vantaggio di essere meno sensibile all'influenza del rumore di fondo.

Il confronto viene effettuato utilizzando simulazioni numeriche della prova e dati sperimentali riguardanti differenti profili di rigidità, riportando le differenze in termini di curva di dispersione sperimentale.

Infine il profilo di rigidità corrispondente all'inversione dei dati relativi alle onde superficiali viene confrontato con i risultati di prove sismiche in foro, evidenziando l'affidabilità dei risultati ottenuti.