Sandy soils parameter identification by means of Simulated Annealing techniques

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Summary

An application of different numerical techniques to inverse problems in geotechnics is presented in this work. Sandy soils are characterized with the hyperbolic model of Ferreira from the load phase of pressuremeter tests carried out in laboratory. The identification of the model parameters is developed with classical optimization and Simulated Annealing techniques. The results obtained with a direct search optimization method within a Simulated Annealing strategy are presented and compared with those obtained with classical techniques type Marquardt with explicit restrictions for the model parameters.

Key Words: Inverse Problem, Simulated Annealing, Geotechnics

1. Introduction

Back analysis of the behaviour of in-situ tests and real structures has always been an important part of the best geotechnical engineering practice[1, 2, 3, 4]. The geotechnic tests characterize the behaviour of the natural medium – drained sands in the present paper – by means of a constitutive model defined by a set of equations and representative parameters of this medium.

The interpretation of drained pressuremeter tests has been the subject of research since the early 60's, when it was presented. A pressuremeter test is usually considered drained or undrained depending on the test speed. However, it is accepted that drained response occurs when pore water is free to move within the soil voids and the mobilized strength is based on the friction of soil particles. The undrained response of the soil during pressuremeter expansion has the consequence of no volumetric change within the soil medium during the test and, that the mean normal stress is considered to remain constant before the stress path reaches the undrained strength surface. Therefore, the radial and circumferential strain increments, measured in cylindrical coordinates, have opposite directions and pure shear condition is considered.

For drained pressuremeter tests the soil response is no longer pure shearing. Additional variables such as volumetric strain and change of the mean normal stress must be taken into account. When the pressuremeter cell is expanded during a drained test, the total volumetric change has two components: compressibility due to the normal stress increase, and dilation or contraction due to the shearing process – dilation angle –.

Baguelin *et al.* [5] studied the error in the estimated friction angle if properties such as softening – strength reduction after peak –, compressibility and dilatancy are not considered during the calculations. While the omission of softening and compressibility cause a fairly small underestimation of the friction angle in loose deposits, omission of dilatant response greatly overestimates the friction angle for dense sands. If dilatancy is not considered during the interpretation of pressuremeter tests in loose deposits, the friction angle is underestimated [6]. Hughes *et al.* [7], Houlsby *et al.* [8] and Manassero [9] developed different approaches to interpret pressuremeter tests with different constitutive laws.

In the present work, an hyperbolic law and a linear volumetric strain relationship are considered [10] to describe and interpret drained pressuremeter tests. In this way the change of volume of the medium is considered by means of a linear relationship between the volumetric and the cavity strains.

The main objective of this paper is to describe and present an automatic parameter estimation methodology that can be applied satisfactory to obtain the best fitting parameters of geotechnical models. In this work, we have adopted the simple Ferreira model, but its use in more developed approaches is undergoing.

The adoption of system identification approaches and optimization techniques has allowed a more systematic and rational approach to the characterization of the soils [11]. Basically, in geotechnical engineering, the denomination of inverse problem or parametric estimation refers to the determi-

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nation procedure of the model parameters – vector of parameters – from the experimental data – pressuremeter test – and prior available information. The inverse analysis procedures are referred to an iterative process of comparison between the experimental data supplied by the pressuremeter test and the results obtained from the numerical model, in order to determine the best set of model parameters. Consequently, a function called objective function is defined from a probabilistic framework based on the concept of maximum likelihood, that represents a conceptual alternative to the more classical Bayesian approach [11]. This function contains the information provided by the two groups of data, measured data and predicted data and is made up by a sum of quadratic terms called errors or residues. Each residue is proportional to the difference between the experimentally measured data and the numerically calculated data. Normally, the process ends when the parameters that minimize locally the value of the objective function are reached.

The automatic minimization of the objective function can be done by different mathematical methods. Basically, there are two ways of searching the minimum: indirect and direct methods [12]. The quasi-Newton methods and the family of Newton methods – Marquardt method – are included, among others, in the first group. In the second group of methods only values of the objective function in different points – set of parameters – are used and do not require either the calculation of the jacobian matrix or of the gradient vector. Generally, these methods converge slower than the indirect search methods although the cost per iteration results much cheaper.

In this paper a direct fitting algorithm based on a Simulated Annealing technique is applied for the first time in pressuremeter analysis. A comparison between both types of minimization methods represented by the Marquardt and the Simulated Annealing methods, considering their advantages and disadvantages, is described in this paper. In addition, an uncertainty analysis of the results is presented.

2. Test description and data employed

It is well known that the pressuremeter test is an extremely useful and economical way for obtaining reliable engineering properties of soil. A typical pressuremeter test response with its load and unload phase is presented in Fig. 1. The pressuremeter test has developed considerably since its first introduction by L. MENARD in 1957. However, the soil disturbance during the pressuremeter installation is one of the main difficulties that still remains, which affects the values of all the parameters deduced from the test [13]. Because of this, the first part of

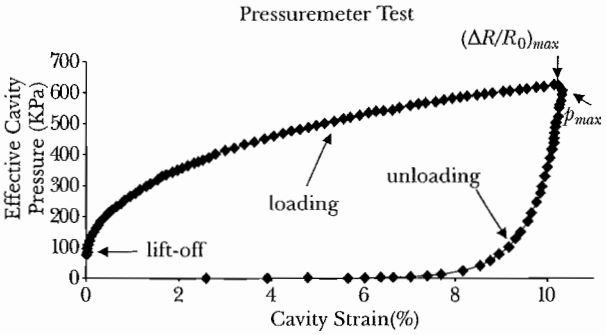

Fig. 1 – Self-boring pressuremeter test response.
Fig. 1 – Risposta sperimentale di un pressometro autoaffondante.

the loading curve usually denotes the disturbance effect. Granular materials are more susceptible to disturbance than frictionless materials. In order to reduce it, the methodology here proposed has been applied to a set of drained self-boring pressuremeter tests carried out in a calibration chamber with Ticino's sand [14]. They were performed under strictly controlled boundary conditions and after a so called ideal installation, where the pressuremeter is placed inside the chamber before filling it with pluvially deposited sand. Only the loading phase of the pressuremeter test has been studied because of the hyperbolic model – Ferreira model [10] – considered. In particular here are presented the results of applying the proposed methodology to two tests, n° 222 and n° 228. However, the same methodology could be applied to the unloading phase if a convenient behaviour model is available.

3. Ferreira Model

In order to characterize the strain-stress response of drained sands from the loading phase of the pressuremeter test a hyperbolic or a non-linear elastic constitutive law proposed by FERREIRA [10] has been selected. The constitutive model has the following assumptions: (I) The pressuremeter test is performed drained during the loading phase. (II) The test is treated as an expansion of an infinitely long cylindrical cavity (i.e., radially symmetric and plane strain). (III) The vertical stress remains the intermediate principal stress (σ_2) during the test. (IV) The soil stress-strain response during loading can be represented by a hyperbolic function. (V) The strains are considered to be small and positive when contraction takes place. The selection of the hyperbolic representation of soil behaviour has been made for the following reasons: (I) The hyperbolic stress-strain model [15] has proved effective in describing soil behaviour under a variety of loading conditions [16], (II) there is a need to keep the soil model simple and to avoid generating a methodol-

ogy that requires a solution for many unknown parameters and (III) the parameters that define the soil model have some engineering significance. On the other hand, the volume variations are taken into account by means of a linear relationship between the volumetric and the cavity strains.

For pressuremeter testing the cavity strain (ε) is often replaced by the circumferential strain (ε_θ).The cavity strains is always the opposite of the circumferential strain: $\varepsilon = -\varepsilon_\theta$. Circumferential strain is derived from Fig. 2 and it has negative sign because it is an expansion,

$$\varepsilon_\theta = \frac{initial\ arch - generic\ arch}{initial\ arch} = \frac{R_o d\theta - R d\theta}{R_o d\theta} = \frac{R_o - R}{R_o} = -\frac{\Delta R}{R_o} \quad (1)$$

Therefore, the cavity strain (ε_θ) is defined as the ratio between the change in pressuremeter radius ($\Delta R = R - R_0$) and the initial pressuremeter radius (R_0). R is the current radius of the pressuremeter.

In this model a linear relationship, Fig. 3, is considered between the volumetric and the cavity strains:

$$\varepsilon_V = -\Delta V/V_o = -s_v \varepsilon \quad (2)$$

where ε_V is the soil volumetric strain, $\Delta V = V - V_0$ is the change in soil volume, V is the current volume, V_o is the initial volume and s_v is the slope of the volumetric strain relationship. s_v is positive during dilation and negative during contraction. Therefore the volumetric strain is positive when contraction takes place.

The stress-strain constitutive relationship here adopted is defined in terms of stress ratio (t/s) as a function of the cavity strain (ε) (Fig. 4):

$$t/s = \frac{\varepsilon}{1/S_i + \varepsilon/(t/s)_{ult}} \quad (3)$$

where t is the shear stress defined as the semidifference between the major (σ_1) and the minor (σ_3) principal stresses, s is the effective mean normal stress defined as the arithmetic average between the major (σ_1') and the minor (σ_3') effective principal stresses, S_i is the initial tangent slope of the hyperbolic model response, (t/s) is the mobilized stress ratio:

$$t/s = (\sigma_1 - \sigma_3)/(\sigma_1' + \sigma_3') \quad (4)$$

and $(t/s)_{ult}$ is the ultimate stress ratio during the loading phase.

If the mobilized shear strength is taken as a function of the cavity strain, there will be a family of hyperbolic curves and the one followed by a soil particle during loading will depend on the level of the effective mean normal stress (σ_m') acting on the soil during the test. The effective mean normal stress is

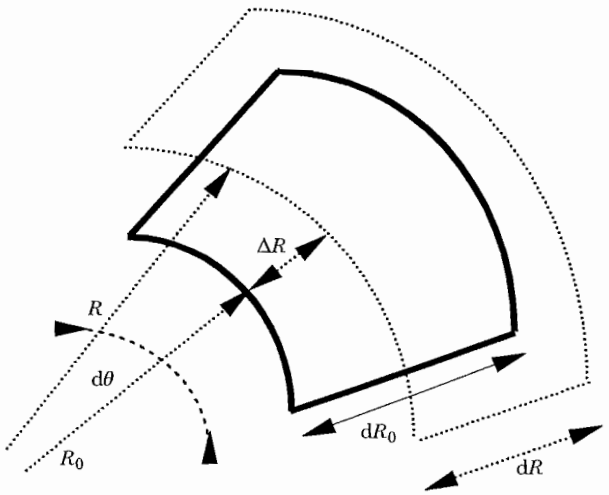

Fig. 2 – Expansion of a cylindrical cavity.
Fig. 2 – Espansione di una cavità cilindrica.

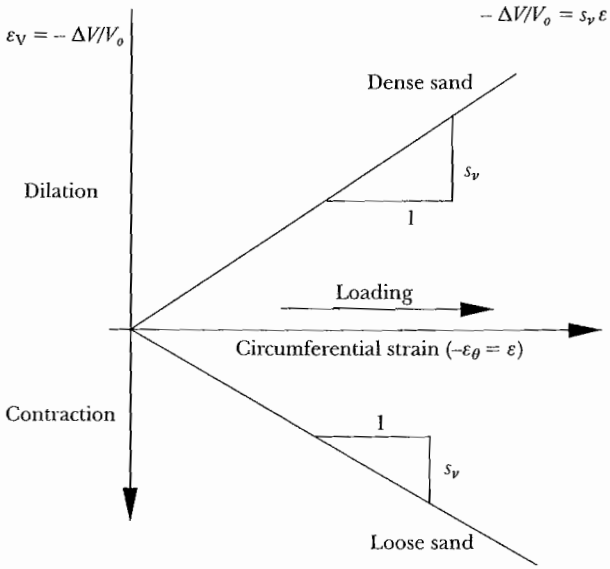

Fig. 3 – Volumetric strain relationship in terms of circumferential strain.
Fig. 3 – Relazione tra deformazione volumetrica e deformazione circonferenziale.

defined as follows: $\sigma_m' = (\sigma_1' + \sigma_2' + \sigma_3')/3 = (\sigma_r' + \sigma_z' + \sigma_\theta')/3$, where $\sigma_1', \sigma_2', \sigma_3'$ are the effective principal stresses and σ_r', σ_z', σ_θ' are the radial, axial and circumferential effective normal stresses. To overcome this complexity, a normalized shear stress is considered [10] and a stress ratio (t/s) is adopted to represent the unique stress-strain curve followed during test.

The boundary condition at the cavity wall is $\varepsilon = 0$, that implies $\sigma_r' = \sigma_{ho}'$. So if $\varepsilon = 0$ the radial effective normal stress (σ_y') coincides with the horizontal initial in situ stress (σ_{ho}').

The equilibrium equation of the system stresses (the system is treated as an expansion of an infinite-

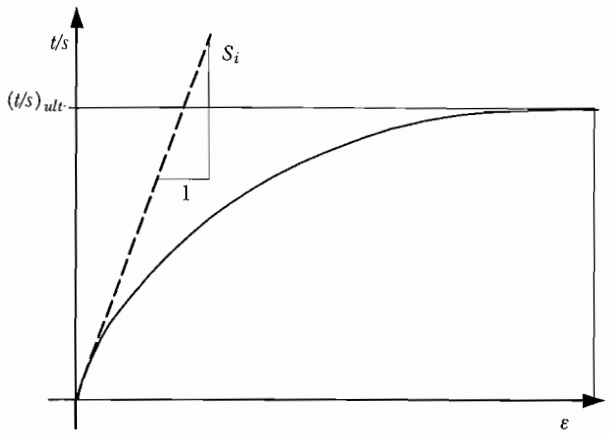

Fig. 4 – Hyperbolic model: stress-strain relationship.
Fig. 4 – Modello iperbolico: relazione tra tensioni e deformazioni.

ly long cylindrical cavity) in terms of effective stresses is:

$$d\sigma_r'/dr + (\sigma_r' - \sigma_\theta')/r = 0 \qquad (5)$$

Combining the hyperbolic soil model (3) with the governing equations of equilibrium of stresses (5), with the compatibility of strains and with the boundary conditions at the pressuremeter cavity wall the following expression of the final pressuremeter analytical loading is obtained [10]:

$$p' = \sigma_{h0}' \left[1 + \left(\frac{1+(t/s)_{ult}}{(t/s)_{ult}}\right) S_i \varepsilon\right]^A e^B \qquad (6)$$

where p' is the effective pressure at the cavity wall of the pressuremeter, $\sigma_r' = \sigma_{ho}'$ is the effective initial in situ stress, e is the basis of the natural logarithm, A is an exponent function of $[S_i, (t/s)_{ult}, K_A^{CV}]$ and B is an exponent function of $[(t/s)_{ult}, K_A^{CV}, \varepsilon]$ defined by:

$$A = \frac{2S_i \left(\frac{1+(t/s)_{ult}}{(t/s)_{ult}}\right) - 2\left(1 - \frac{1-(t/s)_{ult}}{K_A^{CV}(1+(t/s)_{ult})}\right)}{S_i \left(\frac{1+(t/s)_{ult}}{(t/s)_{ult}}\right)^2 \left(1 + \frac{1-(t/s)_{ult}}{K_A^{CV}(1+(t/s)_{ult})}\right)};$$

$$B = \frac{2\left(1 - \frac{1-(t/s)_{ult}}{K_A^{CV}(1+(t/s)_{ult})}\right)\varepsilon}{\left(\frac{1+(t/s)_{ult}}{(t/s)_{ult}}\right)\left(1 + \frac{1-(t/s)_{ult}}{K_A^{CV}(1+(t/s)_{ult})}\right)}$$

where K_A^{CV} is the constant volume friction angle parameter: $K_A^{CV} = (1 - \sin\phi_{CV}')/(1 + \sin\phi_{CV}')$.

From equation (6), the vector of parameters of this hyperbolic model can be written as $\underline{x} = (\sigma_{ho}', (t/s)_{ult}, S_i, K_A^{CV})$, and is used both by the Marquardt type algorithm – indirect method – and by the simplex algorithm with a strategy of Simulated Snnealing – direct method – in the following.

4. Model and soils parameters

The complete loading simulation of drained pressuremeter test proposed by Ferreira depends on four model parameters, $(\sigma_{ho}', (t/s)_{ult}, S_i, K_A^{CV})$. In order to simplify the interpretation procedure, the constant volume friction angle is assumed to be known ($\phi_{CV} \approx 34°$).

The soil parameters derived from the model parameters are the following:

– Initial tangential search modulus (G_i)

$$G_i = S_i \sigma_{ho}'/(2 - \psi) \qquad (7)$$

– Ultimate peak friction angle ($\phi_{ult}' = \phi'$)

$$\phi' = \sin^{-1}(t/s)_{ult} \qquad (8)$$

– Dilation angle (ψ)

$$\psi = sin^{-1}\left(\frac{K_A^{CV} - \frac{1-\sin\phi'_{ult}}{1+\sin\phi'_{ult}}}{K_A^{CV} + \frac{1-\sin\phi'_{ult}}{1+\sin\phi'_{ult}}}\right) \qquad (9)$$

5. Objective function

The objective function has been defined from the maximum likelihood method [17]. This method identifies the probability of the experimental data given the parameters of the model – a hypothesis – with the likelihood of the parameters given the data. That is, the maximum likelihood method looks for the values of the parameters that maximize the likelihood. In other words, the likelihood (L) of the parameters (x) is proportional to the conditional probability of the data (x') given the parameters (x),

$$L(x) = k\, f(x'/x) = k\, P(x) \qquad (10)$$

where k is an arbitrary constant.

When the m data measured x_i^* have independent measure errors, are normally distributed and, furthermore, the variances ($\sigma_i^2 = \sigma^2$) of these normal distributions are equal among them, then the probability of the set of data is equal to the product of the probabilities of each of them,

$$P(x) = \prod_{i=1}^{m} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x_i - x_i^*}{\sigma}\right)^2\right] \qquad (11)$$

where x_i are the correspondent data calculated with the model, and the likelihood function results to be,

$$L(x) = k \prod_{i=1}^{m} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x_i - x_i^*}{\sigma}\right)^2\right] \quad (12)$$

Knowing that, to maximize the likelihood function, is equivalent to maximize its natural logarithm, or equally to minimize its negative natural logarithm. This new function is called support function (S). Calling $\Delta x = k\ 1/\sigma\sqrt{2\pi}$ results that the function that has to be minimized can be written as:

$$S = -2Ln(L) = \sum_{i=1}^{m} \frac{(x_i - x_i^*)^2}{2\sigma^2} - m\ Ln\ \Delta x \quad (13)$$

In the present formulation m, σ and Δx are constant, so the expression (13) coincides with minimizing,

$$f(x) = \sum_{i=1}^{m} (x_i - x_i^*)^2 \quad (14)$$

equation that represents the least squares method. Therefore, when the conditions mentioned are verified the maximization of the likelihood coincides with the classical least squares method [11].

6. Indirect fitting algorithm Marquardt type

One way of determining the parameters of the hyperbolic model ($\underline{x} = (\sigma_{ho}', (t/s)_{ult}, S_t, K_A^{CV})$) is to minimize the value of the objective function $f(x)$, that in this work is made up by a sum of quadratic terms. Each of them is called residue (r) and each residue is defined, in this case, as the difference between the value experimentally measured of the effective pressure (p'^*) and the calculated one (p'). The Marquardt [18] method uses information from the objective function and from its derivatives with respect to each parameter of the model.

$$f(\underline{x}) = \sum_{i=1}^{m} r_i^2(x) = r^t r = (p'^* - p')^t (p'^* - p') \quad (15)$$

One necessary condition to minimize this function is that its gradient be cancelled ($g(x) = 0$) [19],

$$g_j(\underline{x}) = 0 = 2\sum_{i=1}^{m} r_i(\underline{x}) \frac{\partial r_i(\underline{x})}{\partial x_j} = 2\sum_{i=1}^{m} r_i(\underline{x}) J_{ij} \quad (16)$$

where J is the jacobian matrix of m rows (number of data) and n columns (number of parameters x_j) and J_{ij} is the partial derivative of the residue r_i with respect to the parameter x_j . To solve (16) by the Newton method it is also necessary to know the hessian matrix (H),

$$H_{kj} = 2\sum_{i=1}^{m} \left(J_{ij} J_{ik} + r_i(x) Q_{ijk}\right) \quad (17)$$

where H_{kj} is the partial derivative of $g_k(x)$ with respect to the parameter x_j and Q_{ijk} is the second partial derivative of r_i with respect to the parameters x_k and x_j. If the hessian matrix is approximated by the first term of the r.h.s of equation (17) the equations of Gauss-Newton, that allow to determine the maximum minimization direction ($\underline{t}$) of the objective function for the actual parameters, are obtained:

$$J^T J \underline{t} = -J^T \underline{r} \quad (18)$$

where $\underline{t}$ is the difference between the new set of parameters ($\underline{x}'$) and the old one ($\underline{x}$), that is, $\underline{t} = \underline{x}' - \underline{x}$.

The basic idea in which the method proposed by Marquardt is founded is to replace equations (18) by the equations:

$$(J^T J + \lambda K)\underline{t} = -J^T \underline{r} \quad (19)$$

in which the hessian matrix H is approximated by $H' \approx J^T J + \lambda K$, where K is a scaling diagonal matrix and λ a scalar [19], instead of approximating only by $H' \approx J^T J$. In this way the conditioning of the Gauss-Newton equations is improved. Once the direction ($\underline{t}$) is obtained both the new set of parameters (x'_i), that is $\underline{x}' = \underline{x} + \underline{t}$, and the new value of the objective function $f(\underline{x}')$are calculated.

In the work here presented, constraints over the parameters have been imposed affecting the maximum increment allowed along the direction t_i, in order to obtain a new value x', from the old one x_i. The imposed constraints have been explicit ones ($l_{i,\inf} \le x_i' \le u_{i,\sup}$) and they come from the information related with the tests. In this way the derived parameters will be realistic. Moreover, applying this algorithm to particular experiments it has been verified that the better fittings are obtained when all the parameters are in the same range. Otherwise, it could be possible that the information contained in some of the parameters will be lost because the magnitudes combined are very different.

7. Direct fitting algorithm Simulated Annealing type

The denominated Simulated Annealing methods are relatively new [20, 21, 22, 23] and they are employed for solving problems where it is important to determine the global extremes of the objective function. The origin of the Simulated Annealing methodology is in the analogy that it presents with thermodynamic phenomena like the cooling and the following crystallization of the fluids, or the way in which metals get cooled or "annealed".

The molecules of a liquid are moving freely among them when the temperature is high. As the liquid cools and if this cooling is slow, the mobility of the molecules decreases progressively and the atoms get into order. If the process continues a pure crystal completely ordered is obtained. That is, in these conditions a minimum state of energy is reached. On the other hand, if the cooling is quick the final crystal will not be pure and an amorphous polycrystalline formation of higher energy will be obtained. Therefore, it is essential that the cooling process will be slow in order to allow the atoms to loose their mobility and have enough time to arrange themselves in a minimum configuration of energy. This is, technically speaking, the definition of annealing and it is fundamental in order to assure that a minimum state of energy is reached.

Generally, although the analogy is not perfect, it can be said that most of the minimization algorithms advance towards the maximum direction of cooling. In this way, from an initial point – set of parameters –, the quickest and shortest way in the direction of maximum minimization is searched. This approach does not always imply that the global minimum is reached, but in many cases local minima are obtained. The core and the purpose of the Simulated Annealing method are showed in Fig. 5.

METROPOLIS *et al.* [24] were the first who took into account all the considerations from the analogies with thermodynamics and to include them in a numerical formulation. In fact, the algorithm named Metropolis searches for the global minimum of the objective function in the direction of maximum minimization and from time to time advances in the opposite direction.

To work with this algorithm it is necessary to have the following elements:

- A description of the possible configurations of the system to be studied.
- A random generator of configuration changes in the system representing the possible ups and downs of the system.
- The objective function that is going to be minimized.
- A control parameter (T) and an "annealing schedule" describing the way in which the process goes from high energy levels to smaller ones. That is, after how many random changes of configuration a new step of the control parameter (T) in the direction of the maximum minimization – minor value of T – happens and which is its magnitude. In order to understand well and employ rightly this schedule it is necessary to have both the information about the physical behaviour of the system and the information from tests carried out before.

7.1. Application methodology: determination of geotechnical parameters

When the Simulated Annealing methodology is applied to the minimization of the objective function, it is considered that the workspace is continuous n-dimensional, where n represents the number of parameters from which the material behaviour law depends.

In this case, the purpose is to find the global minimum of an objective function ($f(x)$) of type (14). This minimum will be reached with a particular set of parameters (x) representing the state of the system mentioned by Metropolis. The control parameter (T) remains to be something similar to the temperature variable that decreases gradually following the instructions of the annealing schedule. The random generator of both the states of the system and the sets of parameters determine the way of the random advance from a x state to the next step $x+\Delta x$. There are different schemes or possibilities of choosing the new step Δx [25, 26].

The Simulated Annealing method here mentioned [22] is based on a variation of the direct search method called SIMPLEX [27]. Each state of

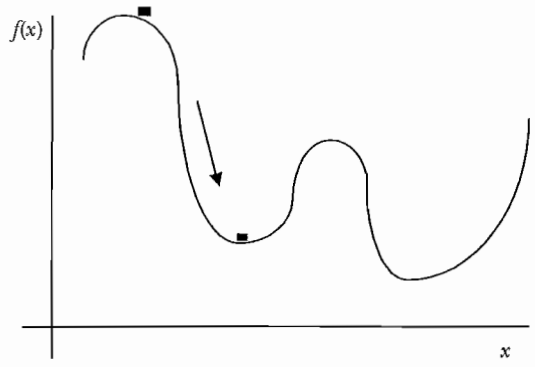

Fig. 5 – Core of the Simulated Annealing technique.

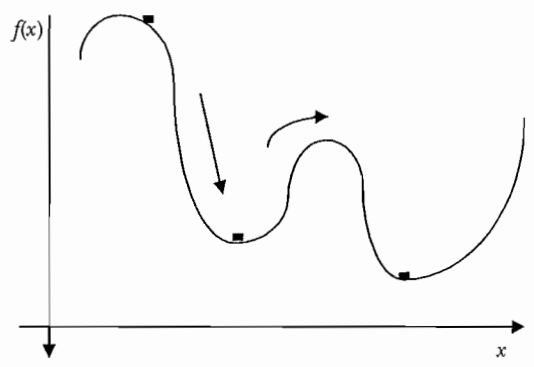

Fig. 5 – Concetto base della tecnica di Ricottura Simulata.

the system or set of parameters (x) generate a simplex formed by $n+1$ vertexes. This simplex, through reflection, contraction and expansion movements advances to the minimum energy state – set of parameters that minimizes the objective function –.

The implementation of the Metropolis algorithm is developed as follows: on one hand, a positive random variable with a logarithmic distribution proportional to the control parameter T is added to the value of the objective function associated to each of the vertexes. On the other hand, a similar random variable is subtracted to the value of the objective function associated to a new state (x) that is tested as a possible point substituting the previous state. In this way, the method always accepts a step in the direction of the maximum minimization, although sometimes it also accepts à step in a different direction. In the limit, when the control parameter tends to zero, the algorithm behaves like the traditional simplex direct search method, advancing in the maximum minimization direction until the convergence in the minimum – global minimum presumed – is reached. Fig. 6 shows the flow chart of the Simplex-Metropolis algorithm.

For any value of T, the simplex is expanded to reach the scale next to the size of the region of the space that can be "represented" by the value of the control parameter. The greater is T the biggest will be the region represented by the simplex in each case. Inside this region a random Brownian movement is produced allowing the testing of randomly chosen new points in it. In this way the efficiency with which a region is explored is independent of its orientation and narrowness. If the cooling (T) is slowly produced the efficiency will be so good that the simplex will contract toward the region that contains the global minimum searched.

8. Parametric dispersion

Once the minimization of the objective function is reached and the set of optimal parameters has been obtained, it is necessary to know the level of goodness of the estimation. That is, what is the variability or uncertainty of the estimated parameters in respect to the true not known parameters. Specifically, the dispersion of the estimated parameters can have an important meaning depending on whether they are going to be employed or not in the decisions to be taken.

In the Marquardt indirect searching method the standard deviation of each parameter has been calculated from the derivative of the objective function with respect to each parameter of the model. On the other side, with the direct searching method based on the Simulated Annealing techniques, and not having these derivatives available, an approximation of the standard deviation has been calculated through the jackknife [28] method.

This method consists of suppressing one or several measured data and of calculating the correspondent parameters. That is, if the *i-th* observation is omitted an estimation of the set of parameter ($\underline{x}_i$) will be obtained. If the total number of observations is m, proceeding in the same way, m different sets of parameters will be calculated. With these, and with the set of parameters resulting ($\underline{x}$) from all the data observed we can obtain an estimation of the variance of the n model parameters,

$$V(\underline{x}) = \frac{(m-1)}{m} \sum_{i=1}^{m} (\underline{x}_i - \underline{x})^2 \qquad (20)$$

where ($\underline{x} - \underline{x}_i$) is the difference between the parameters obtained ($\underline{x}_i$) omitting the *i-th* observation and those obtained ($\underline{x}$) considering all the observed data. The square root of each of the components of $V(\underline{x})$ can be compared, in some way, to the corresponding standard deviation for each parameter of the model obtained through an uncertainty analysis of an indirect method like Marquardt method.

9. Application and results

The methodology proposed in this paper, has been applied to a group of drained pressuremeter tests carried out by the Ismes on Ticino sands [14]. The results corresponding to two tests n° 222 and n° 228 are presented. Figs. 7 and 8 respectively show the experimental curves (Pres. Cav. (Exp.)), obtained with scaling (Pres. Cav. Marq.(esc)) and without scaling (Pres. Cav. Marq.) the parameters through the Marquardt algorithm, and the fitting through the Simulated Annealing algorithm (Pres. Cav. Sim. Ann.) for each of the mentioned tests. The model parameters can be scaled or not. When the parameters are not scaled the information relative to some of them can be lost because very different magnitude numbers are combined, giving erroneous results. On the contrary, when all the parameters of the model are inside the same range of values it is said that the parameters are scaled. Once the analysis ends, it is necessary to undo the scaling done in order to get the set of parameters that minimize the objective function. In general, better results are obtained using scaled parameters.

Tab. I presents both the model parameters and the soil parameters – initial tangent shear modulus (G_i), friction angle (ϕ') and the dilation angle (ψ) – obtained in each fitting from the corespondent parameters of the Ferreira model. It also shows the

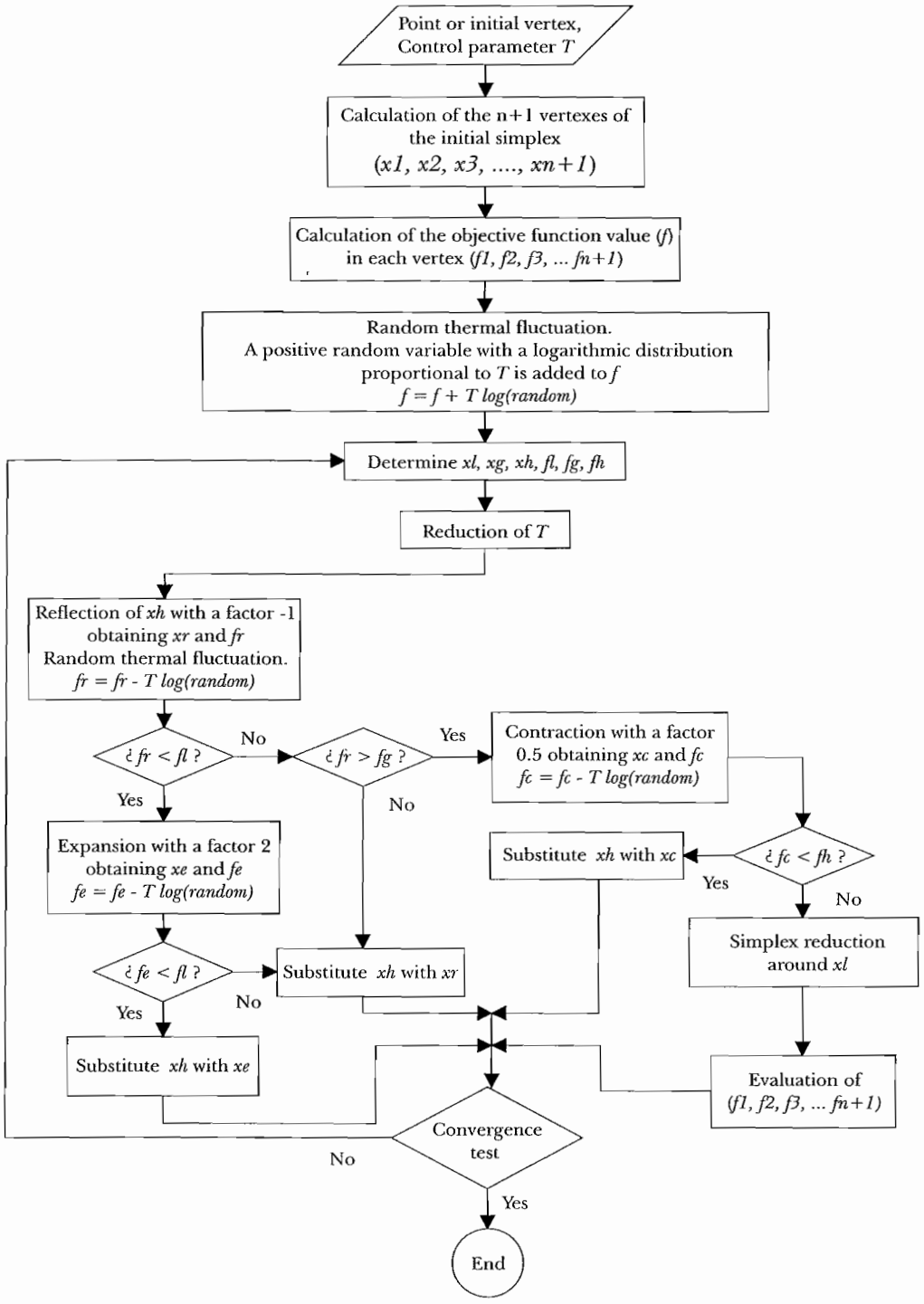

Fig. 6 – Simplex-Metropolis algorithm: f_h the highest objective function value, f_g is the next highest function value and f_l is the lowest function value corresponding respectively to points x_h, x_g and x_l. Furthermore, x_r is the reflected point, x_e is the expanded point and x_c is the contracted point, and f_r, f_e and f_c the corresponding values of the objective function.
Fig. 6 – Algoritmo Simplex-Meropolis: f_h è il più alto valore della funzione obiettivo, f_g il valore alto più vicino a f_h, f_l il valore più basso. Questi valori corrispondono rispettivamente ai punti x_h, x_g, ed x_l. Inoltre x_r è il punto riflesso, x_e il punto espanso e x_c il punto contratto, mentre f_r, f_e, f_c sono i corrispondenti valori della funzione obiettivo.

number of evaluations done of the objective function (N.° Evals.) and the value of the minimum of the objective function (Fun. Obj.). The lower the objective function value is the better the results are. D_R is the sand relative density in each test and (t) is the employed time in seconds. $T=1000$ has been taken as T control parameter in the Simulated Annealing strategy. On the other hand, the K_A^{CV} parameter has been considered almost constant in all simulations.

Good fittings are found, except with the Marquardt fitting without scaling the parameters for test n° 222. Fig. 7 shows how the Marquardt fitting without scaling the parameters in test n° 222 is completely wrong, it does not adjust to the experimental curve. This explains (Tab. I) why the final objective function value is so high (1625770) and why the values of the G_i modulus (4473.4) and dilation angle (-6.041) are out of range compared with those obtained with both the Marquardt algorithm scaling the parameters and the Simulated Annealing algorithm. Fig. 9 represents the evolution of the objective function value versus the number of evaluations correspondent to both tests, with the Simulated Annealing method. The number of evaluations depends on the algorithm employed and on its capacity of minimizing the difference between the experimental and calculated curves.

10. Conclusions

The Simulated Annealing algorithm requires neither the gradient nor the hessian of the objective function, therefore it results cheaper than the Marquardt method traditionally employed in this field. In general, the fittings obtained with the Marquardt method scaling the parameters are much better than those coming from parameters without scaling, although the computational cost is always greater than the employed by the direct method. The fittings reached with the Simulated Annealing technique are good although the parameters have not been scaled. The results obtained with this method without scaling the parameters of the behaviour model of Ferreira are better than those obtained with the Marquardt method. Tab. I shows that both the final value of the objective function and the time employed during the

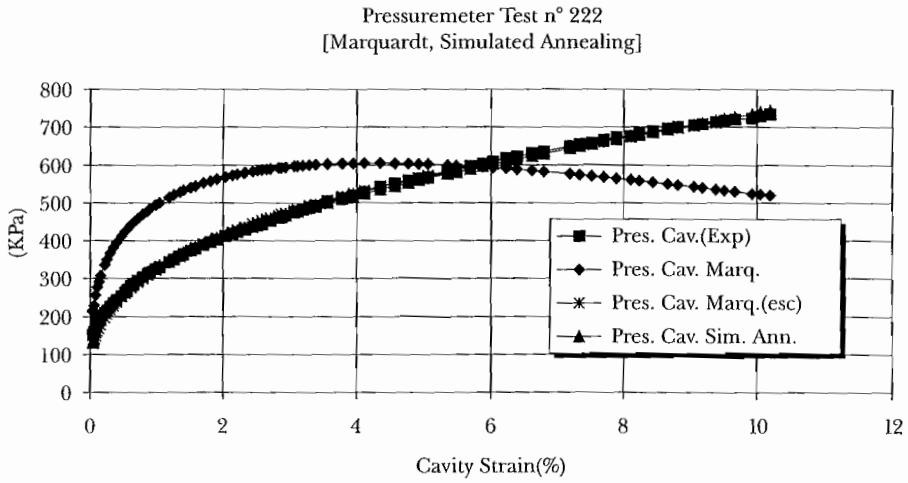

Fig. 7 – Optimization through the Marquardt and Simulated Annealing algorithms. Test n° 222.
Fig. 7 – Ottimizzazione per mezzo degli algoritmi di Marquardt e di Ricottura Simulata. Prova n° 222.

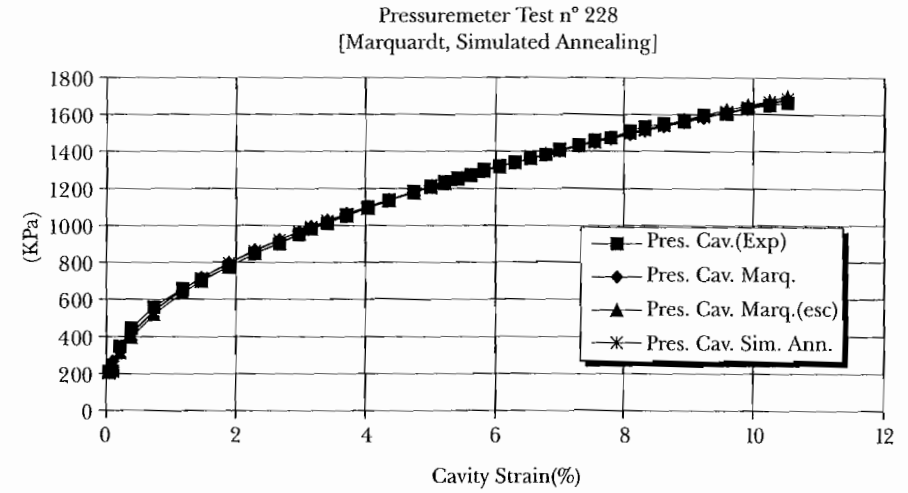

Fig. 8 – Optimization through the Marquardt and Simulated Annealing algorithms. Test n° 228.
Fig. 8 – Ottimizzazione per mezzo degli algoritmi di Marquardt e di Ricottura Simulata. Prova n° 228.

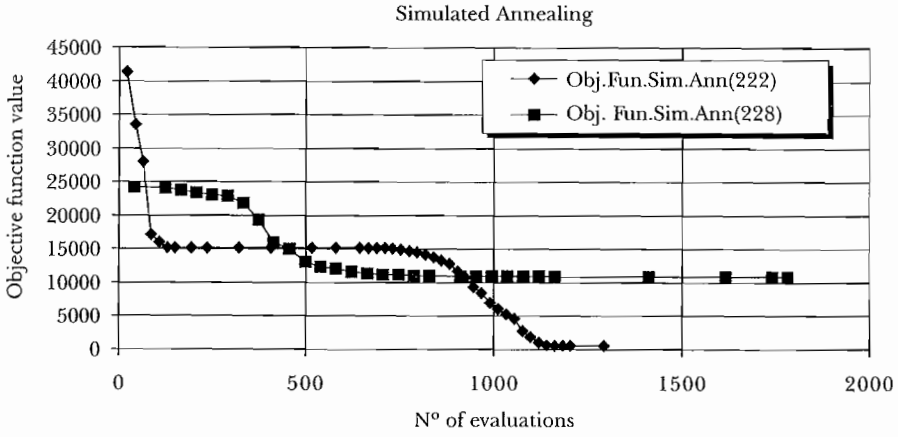

Fig. 9 – Evolution of the objective function versus number of evaluations. Simulated Annealing algorithm. Tests n° 222 and n° 228.

Fig. 9 – Evoluzione della funzione obiettivo con il numero di valuazioni per l'algoritmo di ricottura simulata. Prova n° 222 e n° 228.

Tab. I – Fitted model parameters (σ'_{ho}, $(t/s)_{ult}$, S_i, K_A^{cv}) and soils parameters (ϕ', G_i, ψ).

Tab. I – Valori dei parametri del modello e della legge costitutiva del terreno.

Test	Method	σ'_{ho} (KPa)	$(t/s)_{ult}$	S_i	K_a^{cv}	ϕ' (o)	G_i (KPa)	ψ (o)	N° Evals.	Fun. Obj.	t (s)	D_R [%]
222	S. Annealing	137.88	.56	3.5	.2827	34.05	241.5	0.06	1292	575.2	79	
	Normal Marq.	100	.4823	100	.2827	28.83	4473.4	-6.041	23	1625770	77	46.2
	Scaled Marq.	97.5	.5584	10	.2826	33.94	486.9	-.068	5125	5629.6	5230	
228	S. Annealing	152.06	.63	869.2	.2827	39.05	73307	6.27	1705	10921.9	87	
	Normal Marq.	166.25	.6542	600	.2827	40.85	57345.	8.614	17	54702.4	60	77
	Scaled Marq.	181	.635	600	.2832	39.44	60768.	6.841	1375	65587.8	1234	
	Ferreira	224.4	.653	307.	.2827	40.8	39602.	8.6				
	Hughes					41.9		10				
	Roberston					44.3		13.5				
	Manassero					43.9		12.7				

analyses are lower with the Simulated Annealing method than with the Marquardt method. In Tab. I are also presented the characteristic soil parameters – test n° 228 – obtained in the present work together with those obtained by other authors [7, 9, 29]. It can be appreciated that the parameters obtained with the analysis here presented are slightly smaller. In any case, the interpretation methodology of the pressuremeter test here described is simple and easy to use. Besides, this methodology could be easily applied to other experimental geotechnical tests if a representative model of the behaviour of the tested material [30] is available. For instance, if the soil is modeled by an elastoplastic law and its behaviour is simulated by the finite elements method, the methodology here described can be applied in the same way and with the same advantages [31].

Tab. II shows the results of the uncertainty analysis and how the values of the standard deviation are smaller in the analyses carried out with the Simulated Annealing method than those obtained with the Marquard method. This confirms what has been said here about the utility of the direct method based on the Simulated Annealing techniques.

11. Acknowledgements

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Tab. II – Uncertainty analysis.
Tab. II – Analisi dell'incertezza.

Test	Parameter	Standard Deviationσ
n° 228	σ'_{ho}	184.36
Marquardt	$(t/s)_{ult}$	.104
Scaled	S_i	390.24
Parameters	K_A^{cv}	–
n° 228	σ'_{ho}	15.5
S. Annealing	$(t/s)_{ult}$	3.74 e^{-3}
Not Scaled	S_i	2.04 e^{2}
Parameters	K_A^{cv}	–
n° 222	σ'_{ho}	1.664
S. Annealing	$(t/s)_{ult}$	1.97 e^{-4}
Not Scaled	S_i	1.3 e^{-1}
Parameters	K_A^{cv}	–
n° 222	σ'_{ho}	3.19
Marquardt	$(t/s)_{ult}$	3.67 e^{-3}
Scaled	S_i	0
Parameters	K_A^{cv}	–

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Identificazione dei parametri costitutivi di una sabbia per mezzo di tecniche di ricottura simulata

Sommario

In questo lavoro si presenta un'applicazione di diverse tecniche per problemi inversi in geotecnica. La sabbia è caratterizzata dalla legge iperbolica di Ferreira relativa alla fase di carico di una prova pressometrica. L'identificazione dei parametri costitutivi è condotta mediante una tecnica di ottimizzazione classica e per mezzo della tecnica di ricottura simulata. Questi vengono confrontati con quelli ottenuti con le classiche tecniche di ottimizzazione tipo Marquardt, con restrizioni esplicite sul valore dei parametri del modello.

***Parole chiave**: Problema inverso, Simulazione, Geotecnica.*