

Analysis and design methods of tunnels in squeezing rock conditions

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Summary

The purpose of this paper is to discuss the analysis and design methods of tunnels in squeezing rock conditions. Closed form solutions and numerical analyses are addressed first. Then consideration is given to observation and monitoring during excavation and construction of either the actual tunnel or a test tunnel. Rather than giving theoretical details of the different approaches, emphasis is placed on the solutions which have been developed in order to produce quantitative descriptions of the squeezing behaviour of rock masses, with possible applications to tunnel practice.

1. Introduction

Methods for analysis and design of tunnels in squeezing rock conditions need to consider:

- The onset of yielding within the rock mass, as determined by the shear strength parameters relative to the induced stress
- The time-dependent behaviour.

An additional requirement is the estimate of the support pressure which is able to control the extent of the yielding zone around the tunnel and the resulting deformations. This poses considerable difficulties when the rock mass strength relative to the in situ stress is low and complex support/excavation sequences are envisaged in order to stabilize the tunnel during construction.

It is the purpose of this presentation to address the methods (*closed form solutions* and *numerical analyses*) that are used at the design and analysis stage, with consideration given to the behavioural models which are generally introduced in order to represent the response of the rock mass surrounding the advancing tunnel. In all cases, a word of caution is needed when applying these methods to practical tunnel design in squeezing rock conditions. The difficulty is associated with the assessment of the rock mass properties, as the input data are often not available, inadequate or unreliable.

2. Closed form solutions

The usual approach is to assume the tunnel to be circular and to consider the rock mass subjected to an isotropic initial state of stress, in which the horizontal and vertical stresses are equal. If the attention is paid to the rock mass response to excavation,

which is described by the "rock characteristic line", one can plot the relationship between the support pressure p_1 and the displacement u_r of the tunnel perimeter (Fig. 1).

2.1. Elastoplastic solutions of rock mass response

If the rock mass is assumed to behave as an elastoplastic-isotropic medium, the following models can be used:

- Elastic perfectly plastic (1)
 - Elastoplastic, with brittle behaviour (2)
 - Elastoplastic, with strain softening behaviour (3)
- where, for example, a Mohr-Coulomb yield criterion is introduced (Fig. 1).

A summary of the available closed form solutions for a circular tunnel in an elastoplastic medium is given by BROWN *et al.* [1983], who present a solution where the rock mass follows the Hoek-Brown yield criterion and is considered to dilate during failure. A comprehensive set of solutions of the elastoplastic type has been given by PANET [1995], in his book on the "Convergence - confinement method".

However, if consideration is given to models derived specifically with the squeezing behaviour in mind, the solution due to AYDAN *et al.* [1993] is to be mentioned. As shown in Fig. 2, this solution introduces a four branch stress-strain curve with (i) a linear elastic behaviour up to peak strength, (ii) a perfectly plastic behaviour at peak strength, (iii) a gradual decrease of stress to residual strength with increasing strain, (iv) a perfectly plastic behaviour beyond residual strength.

An example of a typical plot of a characteristic line, under the assumption of elastic-perfectly plastic behaviour, is given in Fig. 3, in conjunction with the thickness of the plastic zone. The tunnel is 11.50m in diameter and is subjected to an in situ stress of 5 MPa. The rock mass failure is defined by the Mohr-Coulomb criterion with a friction angle $\phi = 15^\circ$ and

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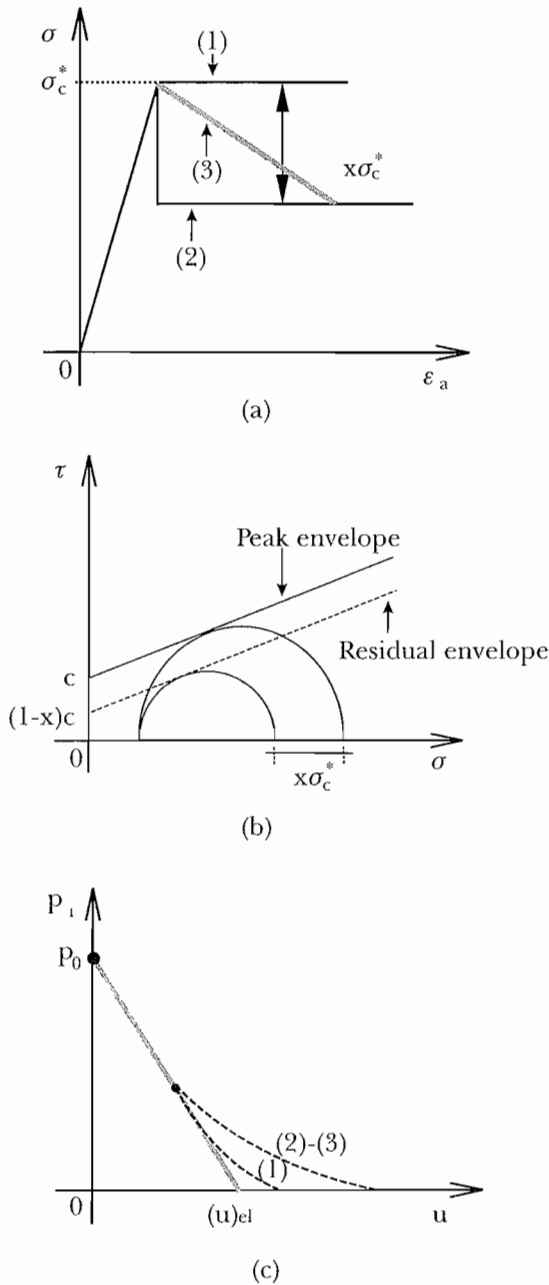


Fig. 1 - Representation of the elasto-plastic models: (a) stress-strain diagrams; (b) yield criterion; (c) characteristic line.

Fig. 1 - Illustrazione dei modelli a comportamento elastoplastico: (a) diagrammi sforzo-deformazione; (b) criterio di snervamento; (c) curva caratteristica.

a cohesive strength $c = 400$ kPa. The rock mass modulus is $E_d = 1.5$ GPa.

2.2. Time-dependent response

The effect of the time-dependent mechanical properties of the rock mass on the response of a tunnel to excavation has been modeled by many authors using viscoelastic and viscoplastic constitutive

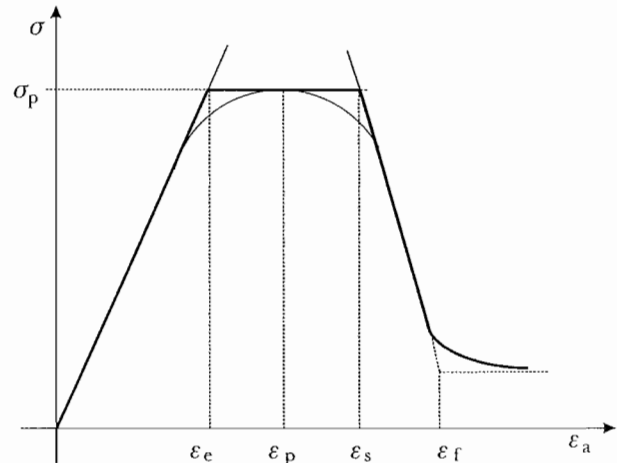


Fig. 2 - Elastoplastic model of squeezing behaviour [AYDAN et al., 1993].

Fig. 2 - Modello elastoplastico di comportamento spingente [AYDAN et al., 1993].

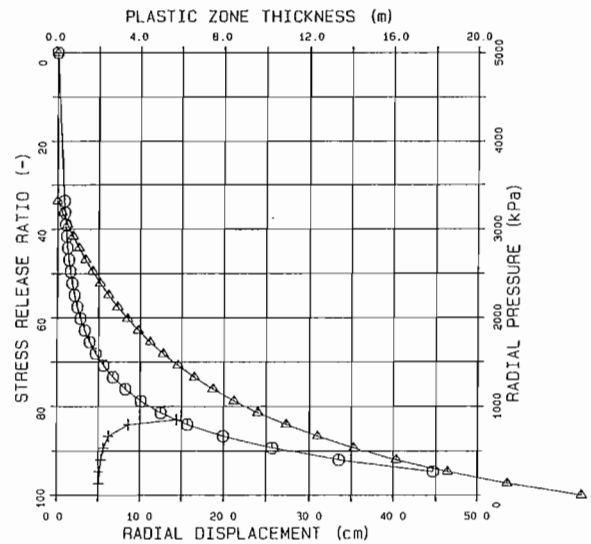


Fig. 3 - Example of "rock characteristic line" plot for a 11.50 m diameter tunnel. The in situ state of stress is isotropic, $p_0 = 5$ MPa. The rock mass failure is defined by the Mohr-Coulomb criterion (cohesion $c = 0.4$ MPa, friction angle $\phi = 15^\circ$) with angle of dilatancy $\psi = 15^\circ$. The in situ deformation modulus $E_d = 1.5$ GPa. Also shown is the "support characteristic line", represented by a 25 cm thick shotcrete lining with steel ribs.

Fig. 3 - Esempio di "linea caratteristica della roccia" per una galleria di 11.50 m di diametro. Lo stato tensionale iniziale è isotropo, $p_0 = 5$ MPa. Il criterio di snervamento assunto è quello di Mohr-Coulomb (coesione $c = 0.4$ MPa, angolo di attrito $\phi = 15^\circ$), con angolo di dilatanza $\psi = 15^\circ$. Il modulo di deformazione $E_d = 1.5$ GPa. E' anche illustrata la "linea caratteristica del sostegno", per un rivestimento in calcestruzzo proiettato e centine metalliche (spessore 25 cm).

laws. LADANYI [1993] and CRISTESCU [1993] give a comprehensive presentation of the available solutions for simple tunnelling cases and models of behaviour:

- Linear viscoelastic
- Linear elastic – linear viscous
- Linear elastic – non linear viscous
- Elastic – viscoplastic.

If consideration is given to squeezing behaviour, the viscoelastic models, where the assumption is that the time effect can be separated from the stress effect in the general creep formulation, are not applicable. Therefore, models of the elastic-viscoplastic type should be used.

A simple model of interest, due to SULEM *et al.* [1987], allows the analysis of time-dependent stress and strain fields around a circular tunnel in a creeping rock mass with plastic yielding. Although valid for a monotonic stress path, this model is well suited for the problem considered and allows a closed form solution for the computation of the time-dependent convergence. As discussed by SULEM [1994], the total strain ε is obtained by adding together the time-independent elastic strain ε^e and the time-dependent inelastic strain ε^{ne} :

$$\varepsilon = \varepsilon^e + \varepsilon^{ne}$$

where: $\varepsilon^{ne} = \varepsilon^p + \varepsilon^c$

for ε^p = plastic strain and ε^c = creep strain.

2.3. Long-term strength concept

The long-term strength concept due to LADANYI [1974; 1993] is often applied in tunnel practice. The implication is that the rock mass properties deteriorate with time, tending to their ultimate long-term values when time tends to infinity (Fig. 4). It may be considered that the use of low bound strength and deformability parameters corresponds to a conservative approach to time-dependent behaviour. How-

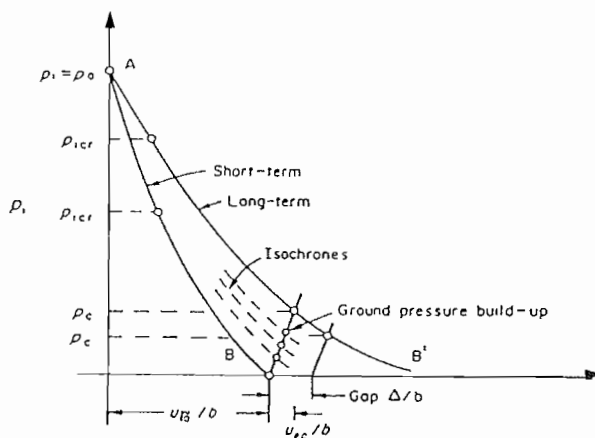


Fig. 4 - "Rock characteristic line" according to the long term strength concept due to LADANYI [1993].

Fig. 4 - "Linea caratteristica della roccia" in accordo all'ipotesi di lungo termine dovuta a LADANYI [1993].

ever, it is obvious that such an approach arbitrarily omits path dependency with all associated induced effects.

In principle, with this approach, all the rock mass parameters are assumed to be time-dependent, but only their limiting values, i.e. short term (peak) and long term (residual) values, are used in the analysis. It should be mentioned that SATO *et al.* [1995] have tried to overcome this difficulty by incorporating in the theoretical model previously developed for circular tunnels (Fig. 3) a set of degradation models for the material characteristics which are assumed to change values with time.

2.4. Rock-support interaction analysis

2.4.1. ROCK MASS RESPONSE

The closed form solutions mentioned above allow one to obtain the "characteristic line" for a circular tunnel and different rock mass response models, under the assumption of isotropy for both the medium and the initial state of stress. These solutions can be very useful in order to gain insights into tunnel behaviour when the excavation takes place in weak rock masses which exhibit squeezing conditions.

As recently shown by HOEK [1998, 1999a]; dimensionless plots can be derived from the results of parametric studies where the influence of the variation in the input parameters has been studied by a Monte Carlo analysis [HOEK, 1998], under the assumption of elastic perfectly plastic behaviour of the rock mass, with zero plastic volume change. Two of such plots are given in Figs. 5 and 6, which were unloaded directly from Dr. Evert Hoek's course notes available on the website: www.rocscience.com [HOEK, 1999a].

Fig. 5 gives a plot of the ratio of the plastic zone radius to tunnel radius and Fig. 6 shows the corresponding ratio of tunnel deformation to tunnel radius versus the ratio of rock mass strength to in situ stress, for the condition of zero support pressure ($p_i = 0$). It is noted that once the rock mass strength falls below 20% of the in situ stress level, the plastic zone size increases very rapidly with a corresponding substantial increase in deformation. It is clear that if this stage is reached, unless the deformations are controlled, collapse of the tunnel is likely to occur.

2.4.2. SUPPORT RESPONSE ANALYSIS

In order to complete the rock-support interaction analysis, the support behaviour is to be considered in detail by determining the "support characteristic line" which relates the confining pressure acting on the support to its deformation. Knowing

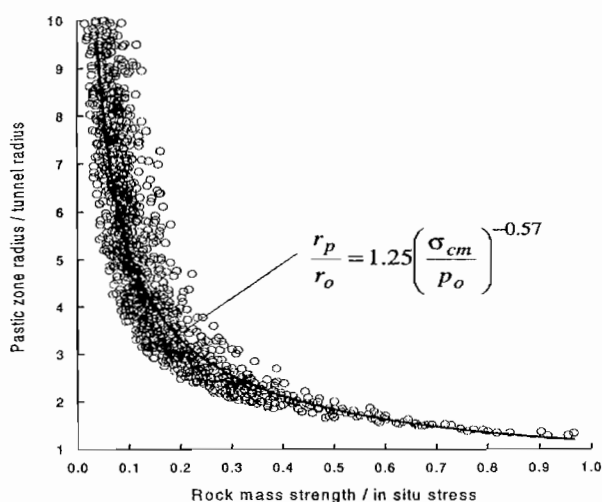


Fig. 5 – Relationship between size of plastic zone and ratio of rock mass strength to in situ stress for weak rock masses [HOEK, 1999b].

Fig. 5 – Relazione tra l'ampiezza della zona plastica e la resistenza dell'ammasso roccioso normalizzata alla tensione in situ per ammassi rocciosi deboli [HOEK, 1999b].

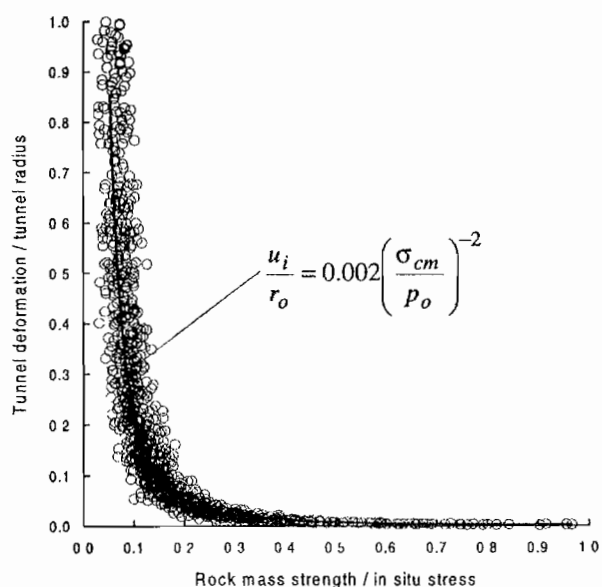


Fig. 6 – Tunnel deformation versus ratio of rock mass strength to in situ stress for weak rock masses [HOEK, 1999b].

Fig. 6 – Deformazione radiale della galleria in funzione della resistenza dell'ammasso roccioso normalizzata alla tensione in situ per ammassi rocciosi deboli [HOEK, 1999b].

the deformation that has occurred before the support is installed at a known distance from the face, the equilibrium solution for the rock support interaction analysis is given by the intersection of the “rock characteristic line” and the “support characteristic line” (Fig. 3).

The “support characteristic line” can be computed by a set of published equations [HOEK and

BROWN, 1980; BRADY and BROWN, 1985] which allow one to determine the stiffness and capacity of the support system. It is to be mentioned that estimates of support capacities for a variety of different systems (steel sets, lattice girders, rock bolts and dowels, concrete and shotcrete linings) for a range of tunnel sizes, have been given by HOEK [1999b]. A word of warning is appropriate by saying that, in all cases, the support is always assumed to act over the entire surface of the tunnel walls, including the invert (i.e. a closed ring condition is always assumed).

The deformation that has occurred before the support is installed is not easy to be determined as complex three-dimensional stress analyses are required in order to account for the influence of the face, the method and sequence of excavation, the possible installation of pre-supports ahead of the face, etc. In simple cases, guidelines have been given by PANET and GUENOT [1982], BERNAUD [1991] and PANET [1995]. In more complex conditions, reference should be made to monitoring results of instrumentation installed before excavation and back analysis, as discussed in the following.

3. Numerical analyses

The use of numerical analyses is advisable in cases when the rock mass strength to in situ stress ratio is below 0.3, and it is highly recommended if this ratio falls below about 0.15, when the stability of the tunnel may become a critical issue [HOEK, 1999a]. Significant advantages are envisaged at the analysis and design stage, when very complex support/ excavation sequences, including pre-support/ stabilisation measures are to be adopted, in order to stabilize the tunnel during construction.

Very powerful computer codes have been developed and are now available for the stress and deformation analysis of tunnels. It is therefore possible to develop reliable predictions of tunnel behaviour, provided a proper understanding of the real phenomena as observed in practice is available. With respect to closed-form solutions, anisotropic in situ stress fields can now be considered, together with multiple excavation stages, the influence of face advance, and the important three-dimensional conditions which occur in the immediate vicinity of the face, the consequence of liner placement delay, etc.

If we remain with the equivalent continuum approach, where the rock mass is treated as a continuum with equal in all directions input data for the strength and deformability properties, which define a given constitutive relation for the medium: elastic, elasto-plastic, elasto-viscoplastic,... the domain methods, which include the finite element

(FEM) and the finite difference (FDM) methods, can be used. An example of a typical stress-deformation analysis of a circular tunnel, for the same properties for the rock mass as shown in Fig. 3, is given in Fig. 7, where the confining pressure p_1 is set equal to 0.8 MPa, which is the equilibrium solution for the rock-support interaction analysis. The results obtained by the FLAC code, compare reasonably well with the closed-form solution as shown in Table I.

One of the obvious advantages of numerical methods in the analysis and design of tunnels in squeezing rock conditions, is the use of more complex laws of behaviour for the rock mass such as the strain-softening and time-dependent behaviour. For example, the elasto-plastic model of Fig. 3 due to AYDAN *et al.* [1993] can be easily implemented with both the FEM and FDM methods. A viscoplastic material law can as well be used as described by GIODA [1982] and Gioda and CIVIDINI [1994], who introduced the rheological model shown in Fig. 8.

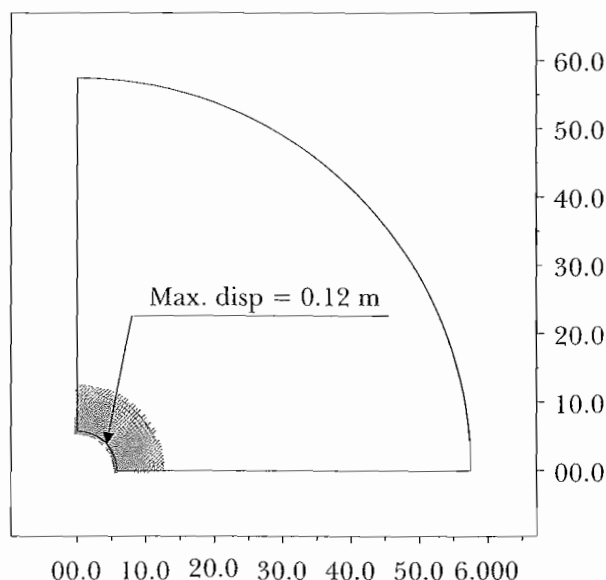


Fig. 7 – Stress deformation analysis of a circular tunnel by the FLAC code. The example shown is described in Fig. 3.
Fig. 7 – Analisi tensio-deformativa di una galleria circolare mediante il codice FLAC. L'esempio è descritto in Fig. 3.

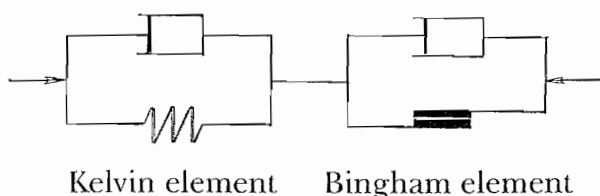


Fig. 8 – Deviatoric rheological model for simulating non-linear creep of squeezing rock masses.
Fig. 8 – Modello reologico di tipo deviatorico per la simulazione della deformazione non lineare di "creep" in ammassi rocciosi in condizioni spingenti.

Tab. I – Comparison of results for characteristic line calculation and FDM solution for the example shown in Fig. 3.

Tab. I – Confronto dei risultati ottenuti con il metodo della linea caratteristica e la soluzione FDM per l'esempio della Fig. 3.

	Characteristic Line	FDM Solution
Radial displacement u_r [cm]	14.00	12.12
Plastic radius R_{pl} [m]	11.75	12.40

This model is composed of a Kelvin model, allowing for primary creep, in series with a Bingham model, allowing for secondary and tertiary creep (Fig. 9).

In weak rock masses which exhibit a squeezing behaviour, the use of continuum representations of the medium subjected to excavation is reasonable. In general, the results obtained are applicable with success in practical tunnel design, provided that engineering judgement and precedent experience are used. However, there are cases where discontinuum modeling could be the most appropriate in order to analyze a given problem.

The example shown in Fig. 10 well illustrates this point of view. The rock mass is argillite, intersected by beddings which strike nearly parallel to tunnel axis. A nearly vertical discontinuity system is as well present. Both the bedding and the jointing are very closely spaced and persistent so that the rock mass is subdivided into very small blocks. The plot shown in Fig. 10 (a) gives a DFN (Discrete Feature Network) model which was created in order to simulate the rock mass behaviour by using the Distinct Element Method and the UDEC code.

The joints are assumed to be Mohr-Coulomb joints, i.e. elastic-perfectly plastic. The blocks are

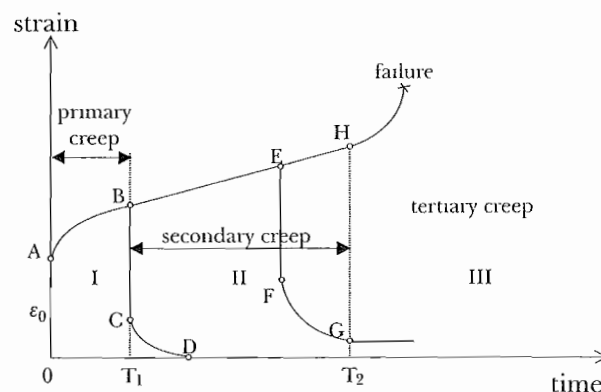


Fig. 9 – Creep deformation stages in a typical strain-time diagram.

Fig. 9 – Fasi del comportamento deformativo di "creep" in un tipico diagramma deformazione tempo.

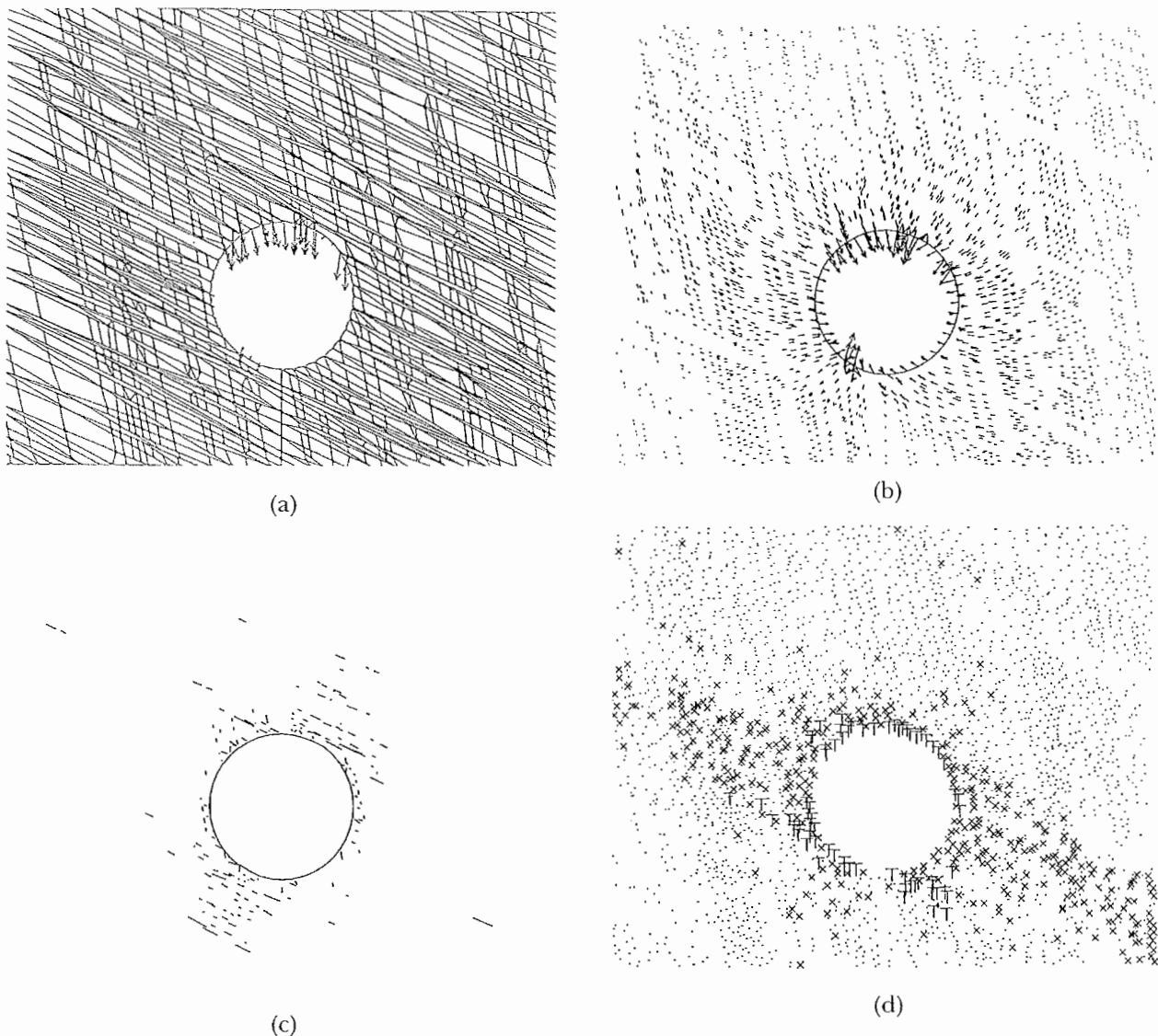


Fig. 10 – Results of a DFN model representing a 11 m diameter tunnel in argillite. (a) DFN model; (b) Displacement distribution (maximum value 0.159 m); (c) Open Zones; (d) Failure Zone.

Fig. 10 – Risultati ottenuti con modello DFN di una galleria di 11 m di diametro in argillite. (a) modello DFN; (b) spostamenti (valore massimo 0.159 m); (c) giunti aperti; (d) zone di plasticizzazione.

treated as an elasto-plastic material which follows the Mohr-Coulomb criterion. The properties of rock blocks and joints are listed in Table II. The in situ state of stress is anisotropic with a vertical stress of 5 MPa and the stress ratio (horizontal to vertical stress) equal to 1.5.

The results obtained are plotted in Fig. 10 (b) to (d) by giving: Fig. 10 (b), the displacement vectors; Fig. 10 (c), the open zones; Fig. 10 (d), the yielded blocks. It is shown that, although the rock mass is very weak and of poor quality, as expected on the basis of the properties above, the orientation of discontinuities plays a very important role on the onset and development of stress and deformations around the tunnel, and therefore also on the squeezing behaviour.

4. Observation and monitoring during excavation

An increased ability to carry out design analyses of tunnels using both closed form solutions and numerical methods as described above is to be recognized. However, the most reliable design approach to deal with squeezing rock conditions is the adoption of performance monitoring during excavation. With this information, the design can be adjusted accordingly. The analysis methods can then be used to great advantage in developing predictions based on the information from supplementary performance monitoring.

As suggested for swelling rocks [ISRM, 1994], also for squeezing rock conditions it is advisable to

Table II – Properties of rock blocks and joints.
 Tabella II – Proprietà dei blocchi di roccia e dei giunti.

Material Properties		Joint properties	beddings	Joints
E [GPa]	15	K_n [GPa/m]	10	10
ν	0.25	K_s [GPa/m]	1	1
c [MPa]	5	c [kPa]	10	50
ϕ [°]	30	ϕ [°]	20	35

use observation and monitoring of a test tunnel (access tunnel, side adit, pilot tunnel, etc.). The test tunnel is to be excavated well in advance of the actual tunnel in order to attain the following:

- identification and quantification of the squeezing behaviour, mainly the ratio of rock mass strength to in situ stress as an indication of the stability conditions of the rock mass surrounding the advancing tunnel;
- in situ observation and monitoring of tunnel convergences and deformations around the tunnel, including the tunnel face and support/pre-support measures;
- comparison of predicted and observed performance in order to improve the computational approach used and to obtain the rock mass parameters to be adopted for final design of the actual tunnel by means of detailed analyses;
- analyze the tunnel response during face advance, by comparing different support measures and excavation sequences, in the attempt to experience either passive or active and intermediate design concepts.

Special care need to be devoted, during performance monitoring of either the actual tunnel or a test tunnel, to the evaluation of the time-dependent behaviour of the rock mass. This is a rather difficult task, especially if one is to determine the basic rock mass creep parameters to be used in the constitutive laws which will be applied for design purposes. Successful examples of application of this type of behaviour have been reported by SULEM *et al.* [1987] and more recently by WITTKÉ *et al.* [1998], who point out that it is essential in squeezing rock conditions to consider in design analyses the time-dependent behaviour of the rock mass.

A common approach is the use of convergence measurements as described by SULEM [1994]. One has, however, to be careful as convergence of a tunnel is due to:

- the effect of the face advance;
- the time-dependent behaviour of the rock mass.

The time-dependent face advance has to be recognized and separated from the time-dependent

rheological behaviour of the rock mass. This is possible by plotting the convergence C versus the distance of the instrumented section to the face of excavation x and versus time t . While the face is close to the considered section, the dominant parameter is the distance to the face and, on the other hand, when the face is far ahead, its influence vanishes and the tunnel response is controlled by the rheological behaviour of the rock mass.

As described by SULEM [1994], a law frequently used to differentiate the face advance effect from the rock mass time-dependent behaviour is written as follows:

$$C(x, t) = Af(x)|1+m \cdot g(t)|$$

where:

$$f(x) = 1 - \left(\frac{X}{x+X} \right)^2$$

$$g(t) = 1 - \left(\frac{T}{t+T} \right)^n$$

for:

X = distance related to the distance of influence of the face (which has been shown to depend on the radius of the yield zone around the tunnel);

T = characteristic time related to the time-dependent properties of the rock mass;

A = “instantaneous convergence” obtained in the case of an infinite rate of advance (no time dependent effect);

$A(1+m)$ = final convergence.

5. Conclusions

A review of current methods for analysis and design of tunnels in squeezing rock conditions has been reported by relying on the work performed by the ISRM Commission on Squeezing Rocks in Tunnels. The attempt has been to describe both the closed form solutions and the numerical analyses that appear to be mostly suited for the design of tunnels in squeezing rock conditions.

It is concluded, with a close eye to applications and present trends in tunnel engineering, that the most reliable approach to deal with squeezing rock conditions is the adoption of performance monitoring during excavation. It is noted that the more complex is a situation, the greater is the need to use a tunnel as an integral part of the design procedure. In all cases, the application of reliable modeling tools (by means of analytical and/or numerical methods) is needed, on one side to match the actual physical process observed in the field, on the other side for design purposes.

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Metodi di analisi progettuale di gallerie in ammassi rocciosi spingenti

Sommario

Lo scopo di questa nota è quello di presentare i metodi di analisi progettuale per gallerie in rocce spingenti. Vengono dapprima richiamate le soluzioni analitiche in forma chiusa ed i metodi numerici di analisi. Quindi si formulano alcune considerazioni sulle osservazioni e misure della galleria in costruzione o di una galleria di prova. La scelta è quella di non entrare nel merito di un dettagliato esame dei procedimenti teorici che caratterizzano ciascun approccio, ma piuttosto di porre l'accento sui metodi di soluzione che sono stati messi a punto per poter descrivere in modo soddisfacente il comportamento spingente degli ammassi rocciosi, avendo sempre ben presente l'applicazione ai problemi reali.